Finite Math - J-term 2017

Lecture Notes - 1/23/2017

Homework

- Section 5.1 - 9, 11, 13, 17, 29, 30, 52, 54

Section 5.1 - Linear Inequalities in Two Variables

Graphing Linear Inequalities in Two Variables. There are 4 types of linear inequalities

$$Ax + By \geq C \qquad Ax + By > C$$

$$Ax + By \leq C \qquad Ax + By < C$$

There is a simple procedure to graphing any of these. If equality is not allowed in an inequality, we call it a *strict inequality*, otherwise we simply call it an inequality.

Procedure.

(1) *Graph the line* $Ax + By = C$ *as a dashed line if the inequality is strict. Otherwise, graph it as a solid line.*

(2) *Choose a test point anywhere in the plane, as long as it is not the line. (The origin,* $(0, 0)$ *is often an easy choice here, but if it is on the line,* $(1, 0)$ *or* $(0, 1)$ *are also easy points to check.)*

(3) *Plug the point from step* (2) *into the inequality. Is the inequality true? Shade in the side of the line with that point. If the inequality is false, shade in the other side.*

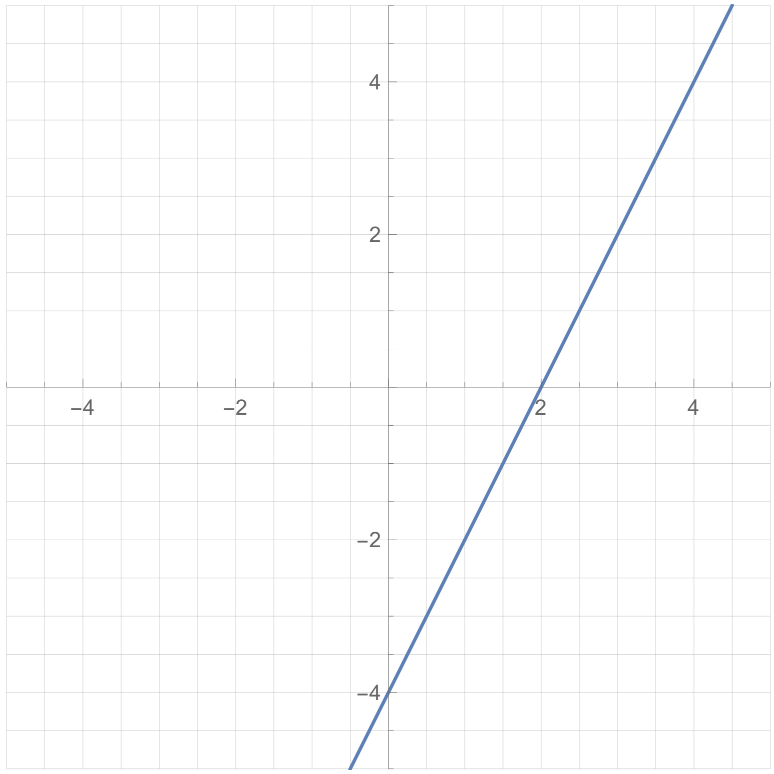
Example 1. *Graph the inequality*

$$6x - 3y \geq 12$$

Solution. *The line we want to graph is*

$$6x - 3y = 12 \quad or \quad y = 2x - 4.$$

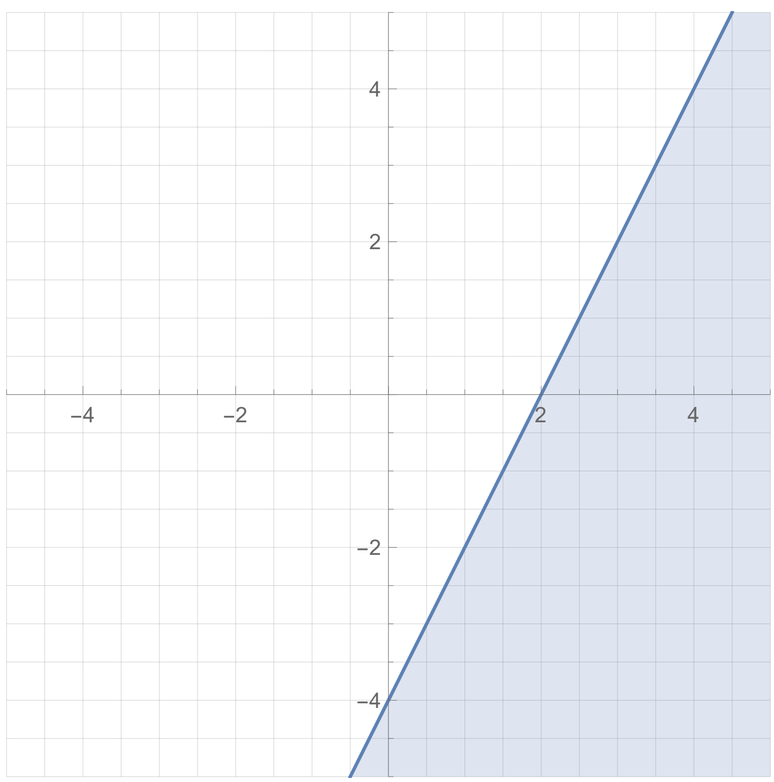
Since the inequality is not strict, we graph it with a solid line.



The point $(0,0)$ is not on the line, so we check that point in the inequality

$$6(0) - 3(0) = 0 \geq 12$$

This is false, so we shade in the side of the line without the origin.



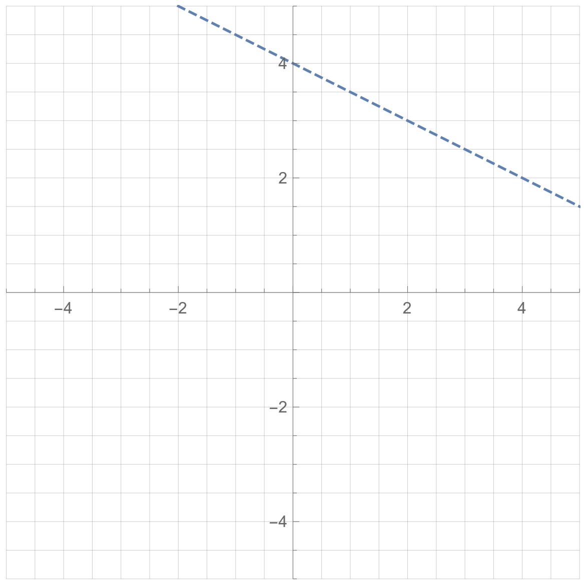
Example 2. *Graph the inequality*

$$4x + 8y < 32$$

Solution. *The line we want to graph is*

$$4x + 8y = 32 \quad or \quad y = -\frac{1}{2}x + 4.$$

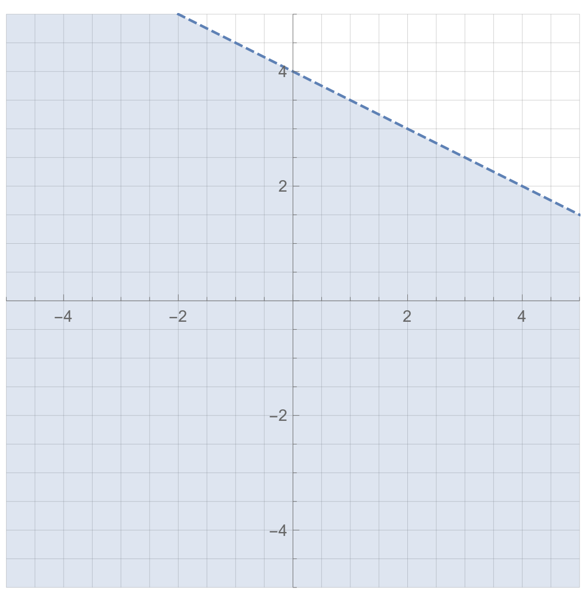
Since the inequality is strict, we graph it with a dashed line.



The point $(0, 0)$ is not on the line, so we check that point in the inequality

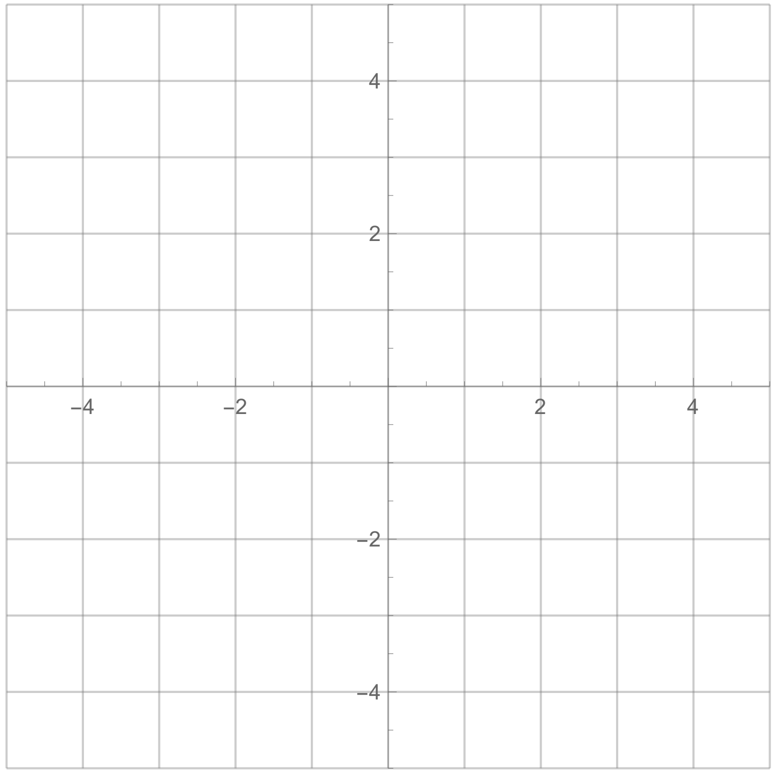
$$4(0) + 8(0) = 0 < 32$$

This is true, so we shade in the side of the line with the origin.



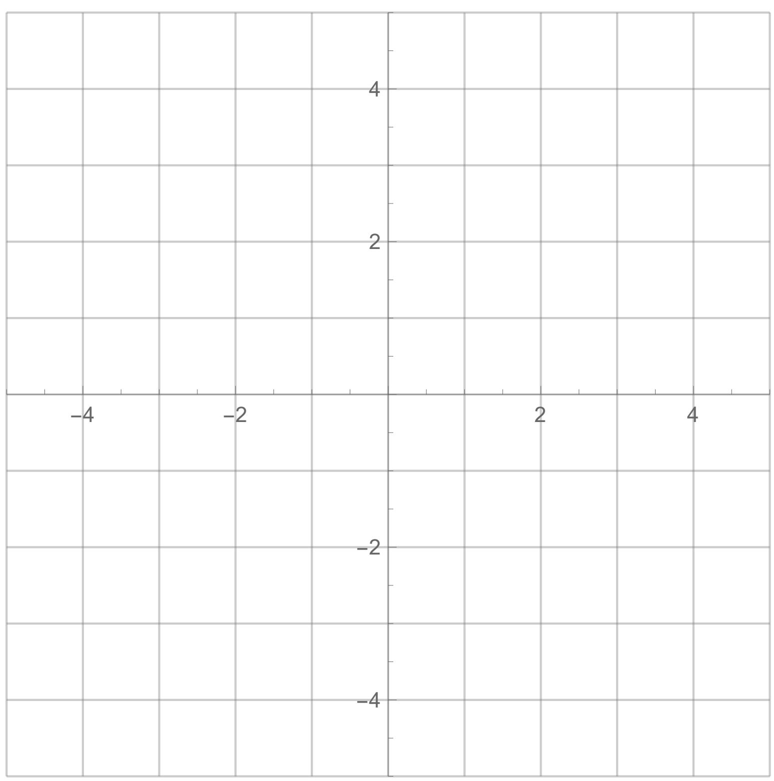
Example 3. *Graph the inequality*

$$2y \leq 10$$

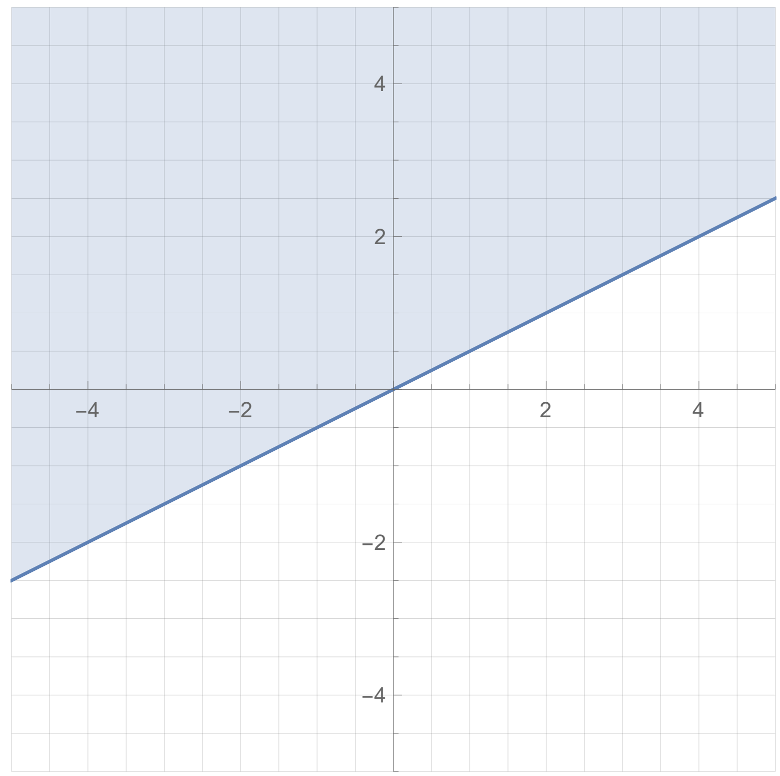


Example 4. *Graph the inequality*

$$2x - 5y > 10$$



Example 5. *Consider the graphed region below.*



(a) *Find an equation for the boundary of the region in the form $Ax + By = C$.*

(b) *Find a linear equality which describes this region.*

Solution.

(a) *Observing the graph, we see that the boundary line passes through* $(0, 0)$ *and* $(2, 1)$. *Using the point-slope form, we get*

$$y - 0 = \frac{1 - 0}{2 - 0}(x - 0)$$

which simplifies to

$$y = \frac{1}{2}x$$

Putting this in the required form gives

$$x - 2y = 0.$$

(b) *Because the boundary line is solid, we are going to replace* $=$ *with either* $\geq$ *or* $\leq$. *To figure out which one, we pick a test point which is not on the line and choose the inequality appropriately. If the test point comes from the shaded region, then we pick the inequality which makes the statement true. If the test point comes from outside the shaded region, pick the inequality*

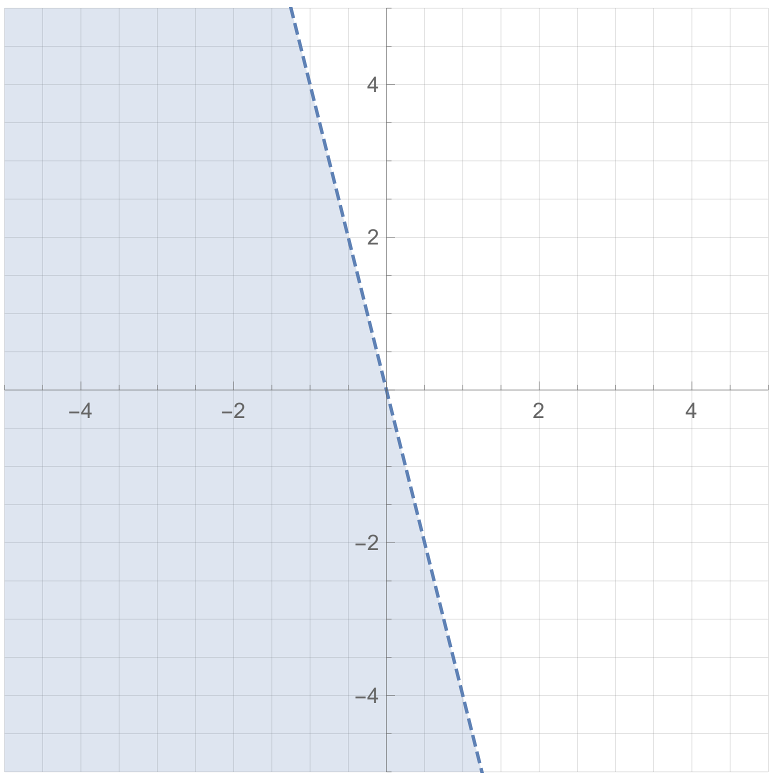
which makes the statement false.
Since $(0,0)$ *actually is on this line, we will pick* $(1,0)$ *as our test point. Notice that* $(1,0)$ *is* outside *the shaded region. Plugging* $(1,0)$ *into the equation gives*

$$x - 2y = 1 - 2(0) = 1 \boxed{?} 0.$$

Since $(1,0)$ *is not in the shaded region, we need to pick the one of* $\geq$ *and* $\leq$ *to replace* $\boxed{?}$ *which makes the statement false. This means we choose* $\leq$ *giving that the inequality for this picture is*

$$x - 2y \leq 0.$$

Example 6. *Consider the graphed region below.*



(a) *Find an equation for the boundary of the region in the form* $Ax + By = C$.

(b) *Find a linear equality which describes this region.*

Applications.

Example 7. *Define the variable and translate the sentence into an inequality:*

(a) *The number of overtime hours is less than* 20.

(b) *Full-time status requires at least* 12 *credit hours.*

Solution.

(a) *Let* h =*number of overtime hours, then* $h < 20$.

(b) *Let* c =*Full-time status, then* $c \geq 12$.

Example 8. *Define two variables and translate the sentence into an inequality: Enrollment in finite mathematics plus enrollment in calculus is less than* 300.

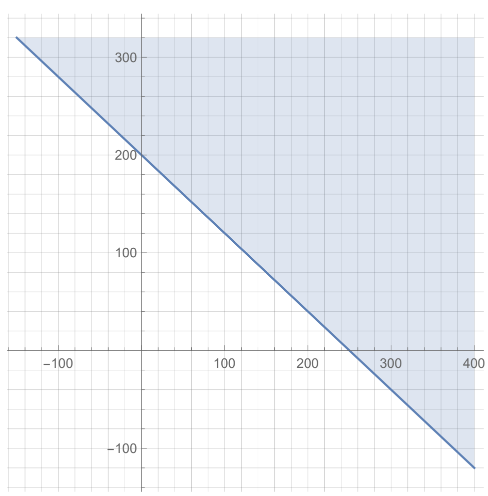
Solution. *Let* F *be the enrollment in finite math and let* C *be the enrollment in calculus. Then* $F + C < 300$.

Example 9. *A food vendor at a rock concert sells hot dogs for* \$4 *and hamburgers for* \$5. *How many of these sandwiches must be sold to produce sales of at least* \$1,000*? Express the answer as a linear inequality and sketch its graph.*

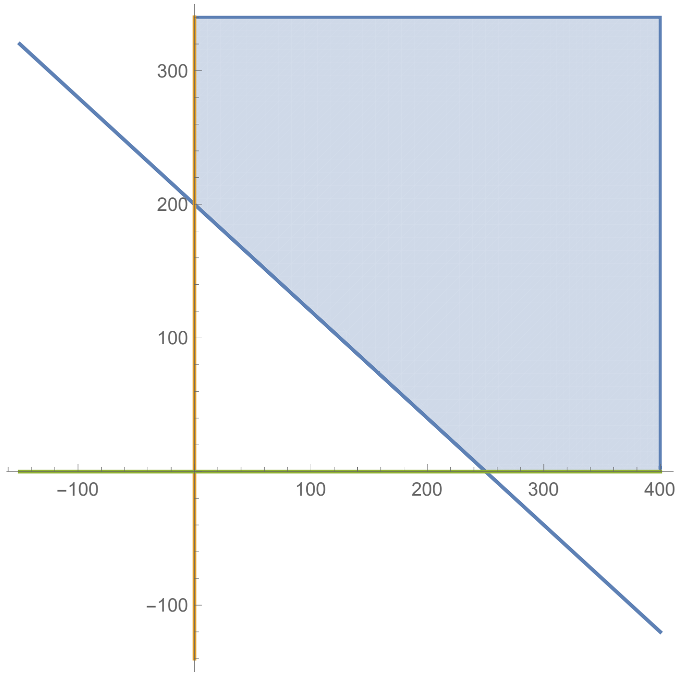
Solution. *Suppose the vendor sells* x *hot dogs and* y *hamburgers. Then the seller has made* $4x + 5y$ *dollars. The sellers wants to make at least* \$1000*, so we get*

$$4x + 5y \geq 1000.$$

If we graph this we get



But since a negative number of hot dogs or hamburgers cannot be sold, we also have the inequalities $x \geq 0$ *and* $y \geq 0$ *to add to this which gives the graph*



The solution is then

$$\begin{cases} 4x + 5y \geq 1000 \\ x \geq 0, \ y \geq 0 \end{cases}$$

Example 10. *Seed costs for a farmer are* \$40 *per acre for corn and* \$32 *per acre for soybeans. How many acres of each crop should the farmer plant if she wants to spend no more than* \$5,000 *on seed? Express the answer as a linear inequality and sketch its graph.*

Example 11. *A farmer wants to use two brands of fertilizer for his corn crop. Brand A contains* 26% *nitrogen,* 3% *phosphate, and* 3% *potash. Brand B contains* 16% *nitrogen,* 8% *phosphate, and* 8% *potash.*

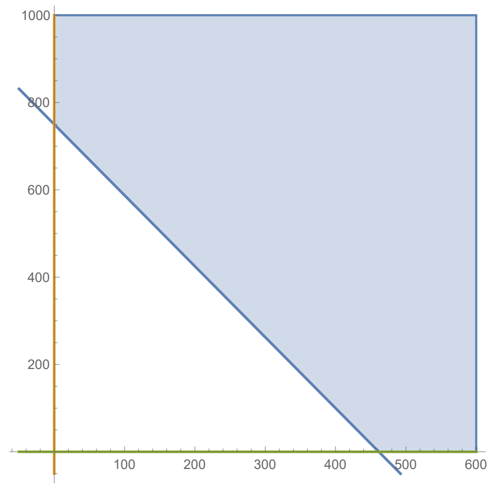
(a) *How many pounds of each brand of fertilizer should he add to each acre if he wants to add at least* 120 *pounds of nitrogen to each acre?*

(b) *How many pounds of each brand of fertilizer should he add to each acre if he wants to add at most* 28 *pounds of phosphate to each acre?*

Solution.

(a) *Let a be the number of pounds of brand A and let b be the number of pounds of brand B. Then*

$$0.26a + 0.16b \geq 120, a \geq 0, b \geq 0$$



(b)

$$0.03a + 0.08b \leq 28, a \geq 0, b \geq 0$$