Finite Math - J-term 2019

Lecture Notes - 1/8/2019

Homework

- Section 2.6 - 65, 67, 68, 69, 70
- Section 3.1 - 9, 11, 15, 16, 18, 20, 22, 24, 26, 29, 34, 39, 50, 55, 58, 63, 71, 79, 81

Section 2.6 - Logarithmic Functions

Using Properties of Logarithms and Exponents.

Example 1. *Solve for x in the following equations:*

(a) $7 = 2e^{0.2x}$

(b) $16 = 5^{3x}$

(c) $8000 = (x-4)^3$

Solution.

A quick reminder of different types of exponents:

- $a^{-n} = \frac{1}{a^n}$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$
  - $a^{1/2} = \sqrt{a}$
  - $a^{1/3} = \sqrt[3]{a}$
- $a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Example 2. *Solve for x in the following equations:*

(a) $75 = 25e^{-x}$

(b) $42 = 7^{2x+3}$

(c) $200 = (2x-1)^5$

Solution.

Applications.

Recall that exponential growth/decay models are of the form

$$A = ce^{rt}.$$

Using the natural logarithm, we can solve for the rate of growth/decay, r, and the time elapsed, t. Let's see this in an example.

Example 3. *The isotope carbon-*14 *has a half-life (the time it takes for the isotope to decay to half of its original mass) of* 5730 *years.*

(a) *At what rate does carbon-*14 *decay?*

(b) *How long would it take for* 90% *of a chunk of carbon-*14 *to decay?*

Section 3.1 - Simple Interest

Suppose you make a deposit or investment of P dollars or you take out a loan of P dollars. The amount P is called the *principal.*

All of these things have an *interest rate* attached to them, essentially rent on the money, which is paid as *interest.*

Simple Interest. Simple interest is computed as

where I = interest, P = principal, r = annual simple interest rate (written as a decimal), and t = time in years.

Example 4. *Suppose you deposit* \$2,000 *into a savings account with an annual simple interest rate of* 6%. *How much interest will accrue after* 6 *months?*

Solution.

Future Value. Often, we might be more curious about how much will be in the account or how much will be owed on the loan after a certain period. This amount is called the *future value.* Another name for principal is *present value.* It is found by simply adding the original investment/loan amount to the interest accrued.

Definition 1 (Future Value)**.**

and in a simplified form

where A =future value, P =principal/present value, r = annual simple interest rate, t = time in years.

Example 5. *Suppose you take out a* \$10,000 *loan at a simple annual interest rate of* 3.2%. *How much would be due on the loan after* 10 *months?*

Solution.

Example 6. *You make an investment of* $\$3,000$ *at an annual rate of* 4.5%. *What will be the value of your investment after* 30 *days? (Assume there are* 360 *days in a year.)*

Solution.

We can also use the formulas to predict what interest rate we need or how much principal to take out/deposit.

Example 7. *You're looking to invest* $\$5,000$ *and make* $\$100$ *in interest after* 10 *weeks. What annual rate on your investment will you need to accomplish this?*

Solution.

Example 8. *You invest* $\$4,000$ *at an annual rate of* 3.9%. *How long will it take for the investment to be worth* $\$5,000$*? Give your answer in years, correct to* 2 *decimal places.*

Solution.

One often uses a brokerage firm when making investments, many of which charge you a fee based on the transaction amount (principle) when both buying AND selling stocks.

Example 9. *Suppose a brokerage firm uses the following commission schedule*

Principal	*Commission*
Under $3,000	*$25+1.8% of principal*
$3,000 - $10,000	*$37+1.4% of principal*
Over $10,000	*$107+0.7% of principal*

An investor purchases 450 *shares of a stock at* $21.40 *per share, keeps the stock for* 26 *weeks, then sells the stock for* $24.60 *per share. What was the annual interest rate earned on the investment?*

Solution.

Example 10. *Suppose a brokerage firm uses the following commission schedule*

Principal	*Commission*
Under $3,000	*$32+1.8% of principal*
$3,000 - $10,000	*$56+1% of principal*
Over $10,000	*$106+0.5% of principal*

An investor purchases 75 *shares of a stock at* \$37.90 *per share, keeps the stock for* 150 *days, then sells the stock for* \$41.20 *per share. What was the annual interest rate earned on the investment? (Again, assume a* 360*-day year.)*

Solution.

Average Daily Balance.

A common method for calculating interest on a credit card is to use the *average daily balance method.* As the name suggests, the average daily balance is computed, then the interest is computed on that.

Example 11. *A credit card has an annual interest rate of* 19.99% *and interest is calculated using the average daily balance method. If the starting balance of a* 30*-day billing cycle is* \$523.18 *and purchases of* \$147.98 *and* \$36.27 *are posted on days* 12 *and* 25*, respectively, and a payment of* \$200 *is credited on day* 17*, what will be the balance on the card at the start of the next billing cycle?*

Solution.

Example 12. *A credit card has an annual interest rate of* 19.99% *and interest is calculated using the average daily balance method. If the starting balance of a* 28*-day billing cycle is* \$696.21 *and purchases of* \$25.59, \$19.95, *and* \$97.26 *are posted on days* 6, 13, *and* 25, *respectively, and a payment of* \$140 *is credited on day* 8, *what will be the balance on the card at the start of the next billing cycle?*

Solution.

Section 3.2 - Compound and Continuous Compound Interest

Compound Interest. In the case of simple interest, the interest is computed exactly once: at the end. Typically, however, interest is usually compounded something like monthly or quarterly.

Example 13. *Suppose* \$5,000 *is invested at* 12%, *compounded quarterly. How much is the investment worth after* 1 *year?*

If we generalize this process, we end up with the following result

Definition 2 (Compound Interest).

The variables in this equation are

- $A =$ *future value after* n *compounding periods*
- $P =$ *principal*
- $r =$ *annual nominal rate*
- $m =$ *number of compounding periods per year*
- $i =$ *rate per compounding period*
- $n =$ *total number of compounding periods*

Alternately, one can reinterpret this formula as a function of time as

where A, P, r, and m have the same meanings as above and t is the time in years.

Example 14. *If* $\$1,000$ *is invested at* 6% *interest compounded (a) annually, (b) semiannually, (c) quarterly, (d) monthly, what is the value of the investment after* 8 *years? Round answers to the nearest cent.*

Solution.