Linear and Polynomial Functions

Math 130 - Essentials of Calculus

11 September 2019

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Given the slope of a line, m, and a point it passes though (x_1, y_1), an equation for the line is

$$y - y_1 = m(x - x_1).$$

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Example

The weekly ratings, in millions of viewers, of a recent television program are given by $L(w)$, where w is the number of weeks since the show premiered. If L is a linear function where $L(8) = 5.32$ and $L(12) = 8.36$, compute the slope of L and explain what it represents in this context. Write a formula for $L(w)$.

Now You Try It!

Example

The monthly cost of driving a car depends on the number of miles driven. Lynn found that in May is cost her $380 *to drive* 480 *miles and in June it cost her* $460 *to drive* 800 *miles.*

1. *Express the monthly cost* C *as a function of the distance driven* d*, assuming that there is a linear relationship.*
2. *Use part 1 to predict the cost of driving* 1500 *miles per month.*
3. *What does the slope represent?*

POLYNOMIALS

A function P is called a *polynomial* if it can be written in the form

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0 \qquad a_n \neq 0$$

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where n is a nonnegative integer. The numbers $a_0, a_1, a_2, \ldots, a_n$ are called the *coefficients* of the polynomial. The value of the largest exponent n is called the *degree* of the polynomial. The domain of any polynomial is always $\mathbb{R} = (-\infty, \infty)$. ($\mathbb{R}$ is the symbol we often use to denote "all real numbers.")

Quadratic Functions

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Example

Graph the following quadratic functions.

1. $f(x) = \frac{1}{2}(x - 1)^2 - 2$
2. $C(x) = -2(x - 3)^2 - 4$

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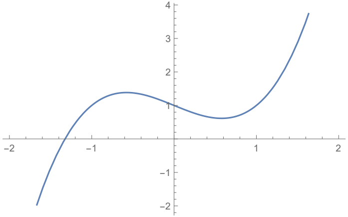

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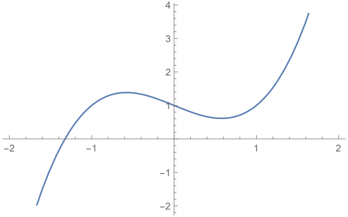

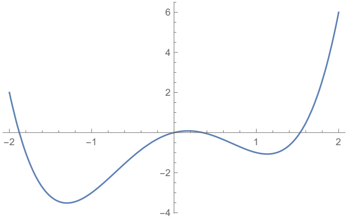

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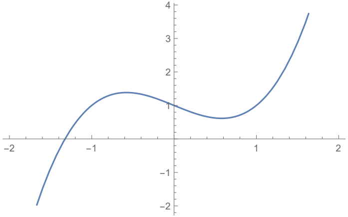

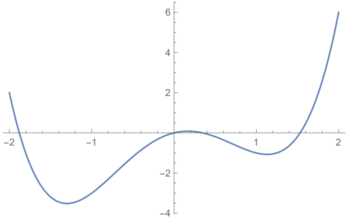

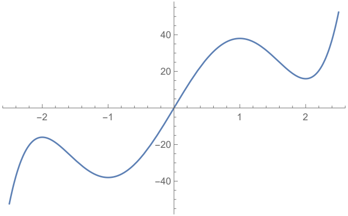

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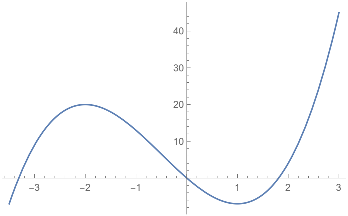

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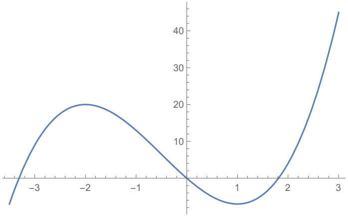

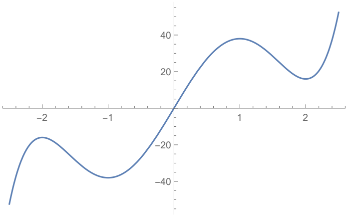

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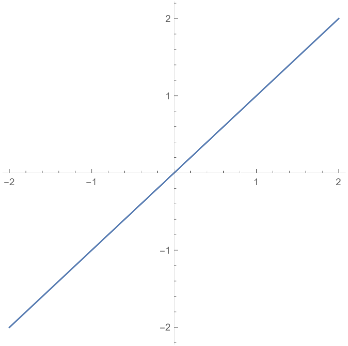

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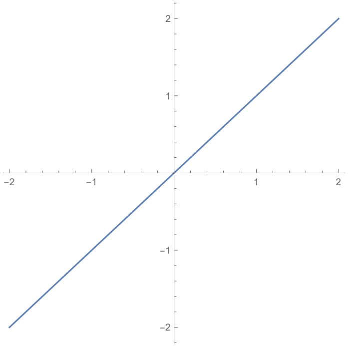

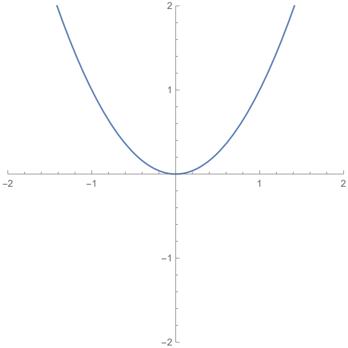

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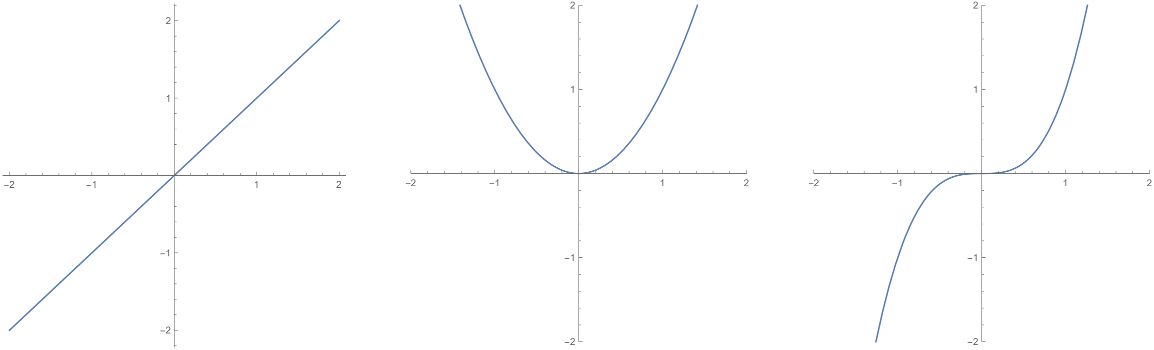

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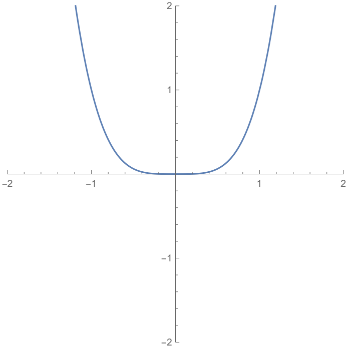

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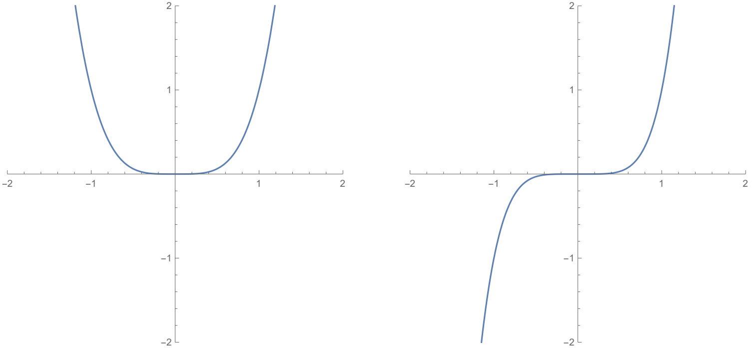

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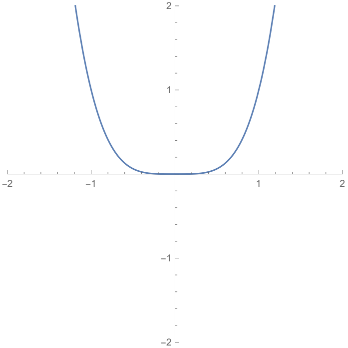

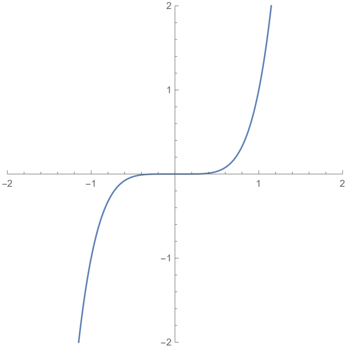

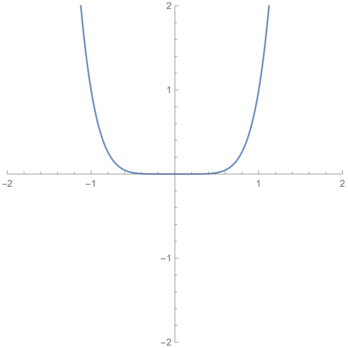

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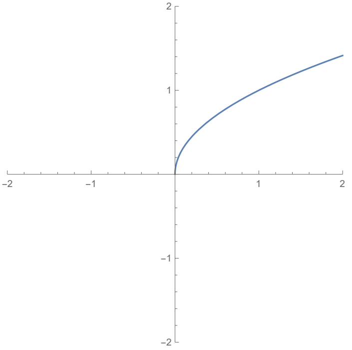

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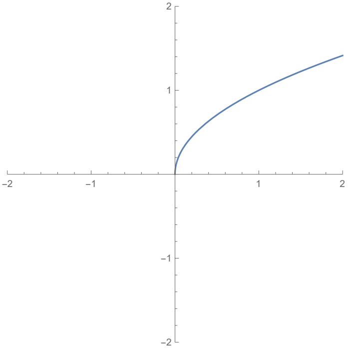

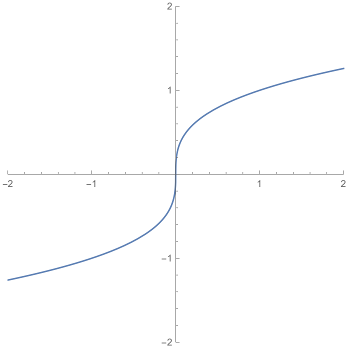

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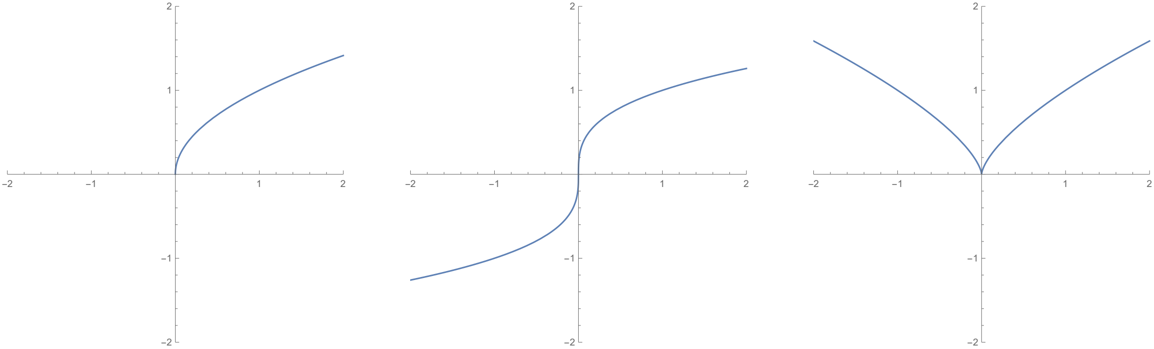

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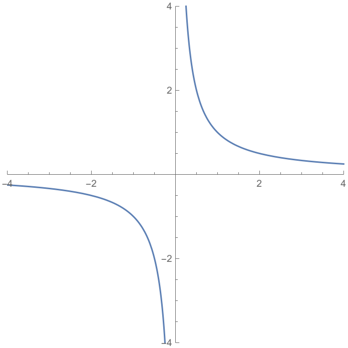

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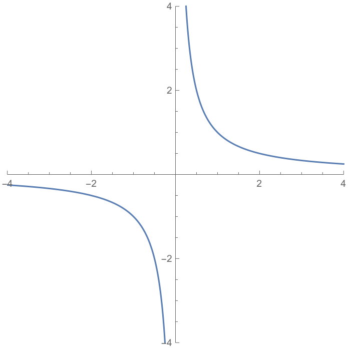

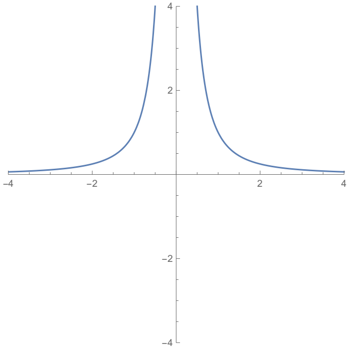

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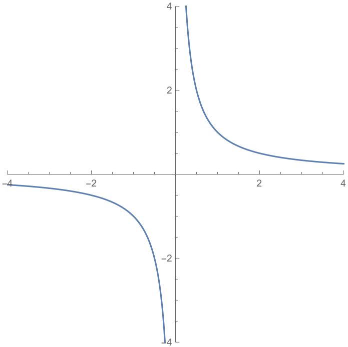

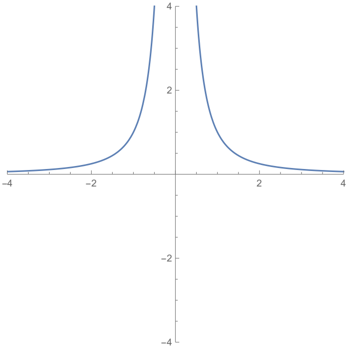

$f(x) = \dfrac{1}{x}$ is also known as the *reciprocal function*.