Polynomial Functions

Math 130 - Essentials of Calculus

13 September 2019

Increasing and Decreasing

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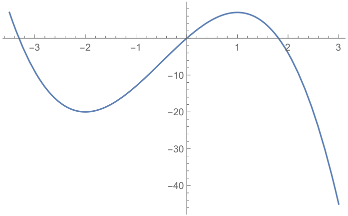

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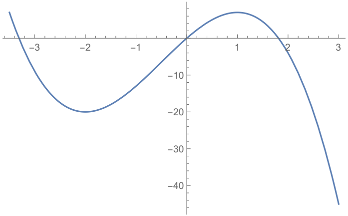

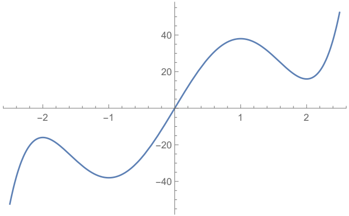

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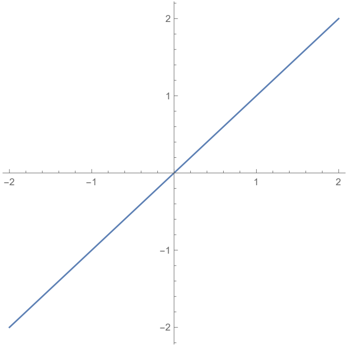

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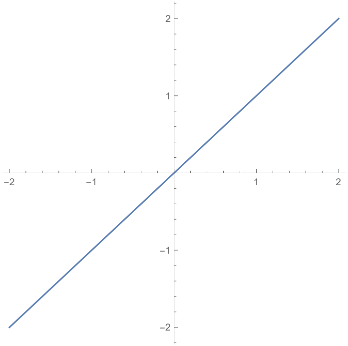

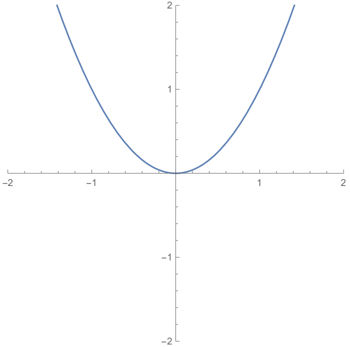

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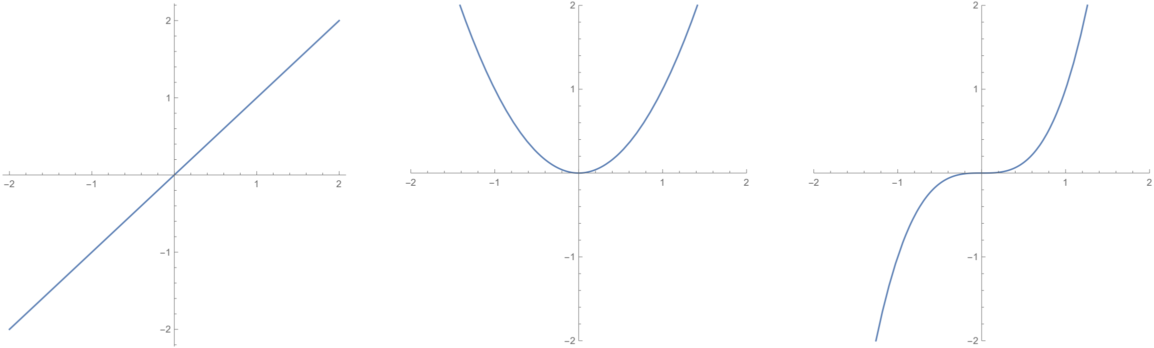

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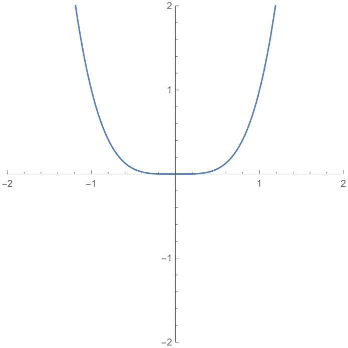

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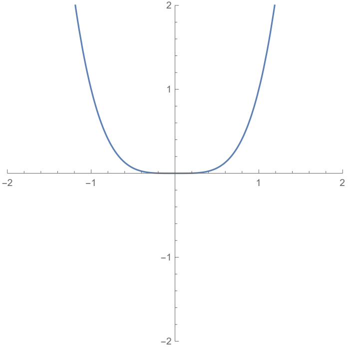

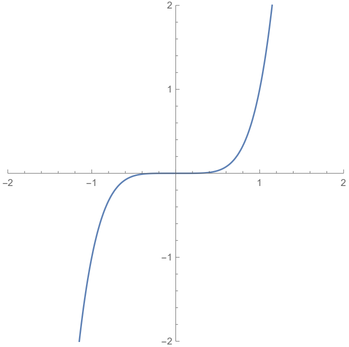

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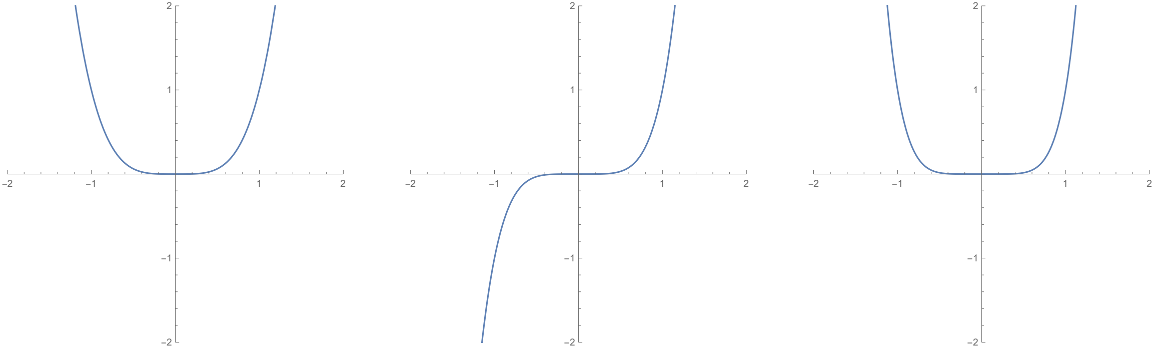

Other Power Functions

Below are graphs of $f(x) = x^{1/2} = \sqrt{x}$, $f(x) = x^{1/3} = \sqrt[3]{x}$, and $f(x) = x^{2/3} = \sqrt[3]{x^2} = (\sqrt[3]{x})^2$

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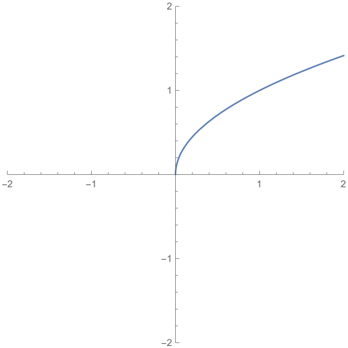

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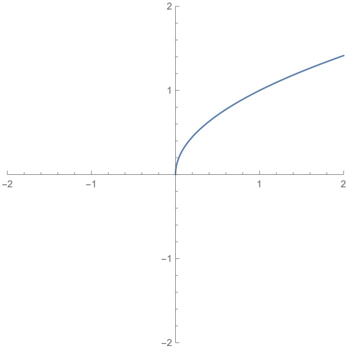

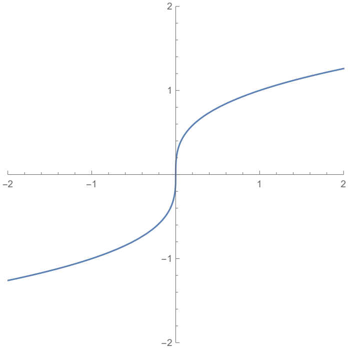

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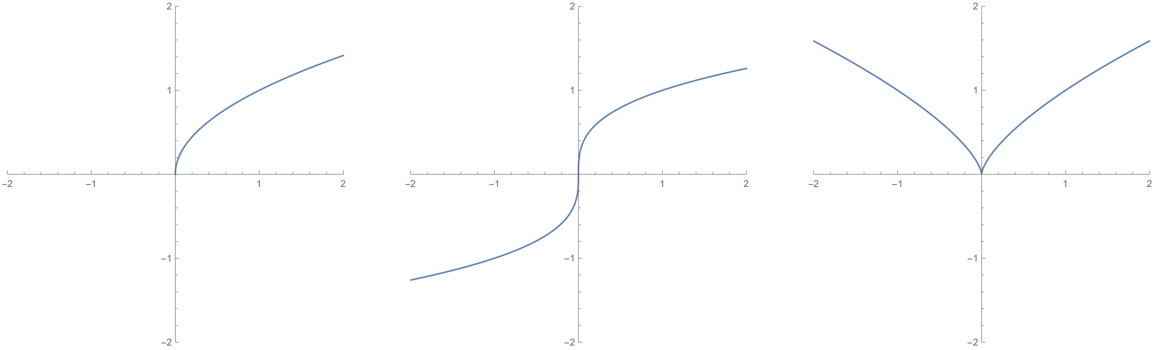

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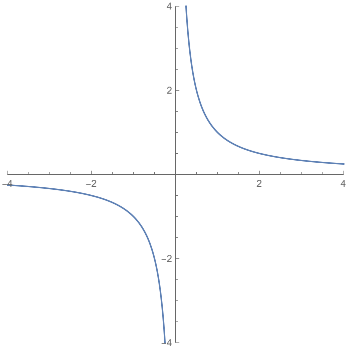

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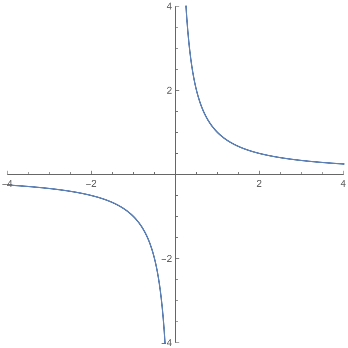

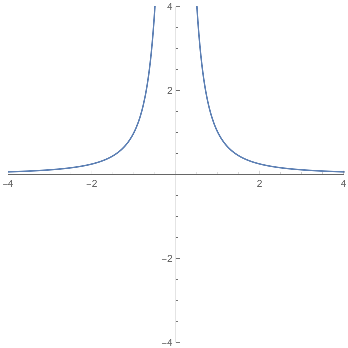

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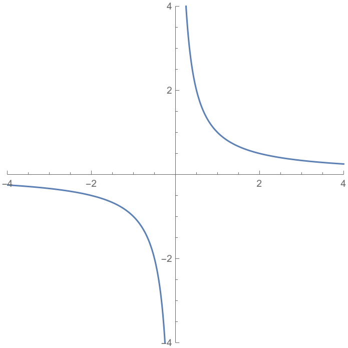

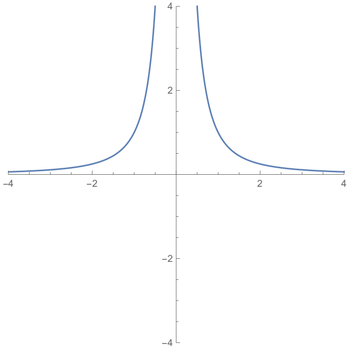

$f(x) = \frac{1}{x}$ is also known as the *reciprocal function*.

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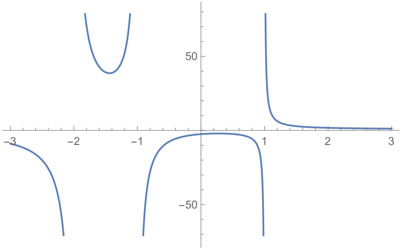
Here is the graph of a rational function:



Proportionality

In mathematical modeling, we often say two quantities are *proportional* or *inversely proportional*.

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From this, we can write the balance as a function of the number of quarters as

$$B(q) = \$1000(1 + 0.02)^q = \$1000(1.02)^q.$$

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4. $(ab)^x = a^x b^x$

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Under ideal conditions a certain bacteria population is known to double every 3 *hours. Suppose that there are initially* 100 *bacteria.*

1. *What is the size of the population after* 15 *hours?*
2. *What is the size of the population after* t *hours?*
3. *Estimate the size of the population after* 20 *hours.*
4. *Graph the population function and estimate the time for the population to reach* $50,000$.

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A very convenient number to take as the base of an exponential function is the natural number e (which has value approximately 2.71828). The function $f(x) = e^x$ turns out to have some really nice properties in terms of calculus (for example, the slope of the tangent line to the graph at $(0, 1)$ is 1. One way we can define the number e is as the value the function

$$\left(1 + \frac{1}{x}\right)^x$$

get closer and closer to as x gets larger and larger.