One Sided Limits and the Derivative

Math 130 - Essentials of Calculus

27 September 2019

More Limits

Example

Compute the limits

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3. $\lim_{x \to 4} \frac{\frac{1}{4}+\frac{1}{x}}{4+x}$
4. $\lim_{x \to 0} \frac{1}{x^2}$

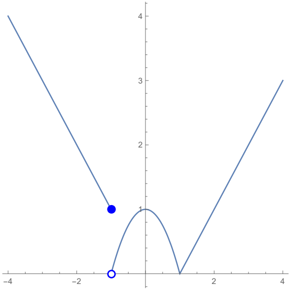
One-Sided Limits

Example

Consider the function

$$f(x) = \begin{cases} -x, & x \le 1 \\ 1 - x^2, & -1 < x < 1 \\ x - 1, & x > 1 \end{cases}$$

Compute the following limits:



1. $\lim_{x \to 1^+} g(x)$
2. $\lim_{x \to 1} g(x)$
3. $\lim_{x \to 0} g(x)$
4. $\lim_{x \to -1^-} g(x)$
5. $\lim_{x \to -1^+} g(x)$
6. $\lim_{x \to -1} g(x)$

Relation Between One-Sided and Two-Sided Limits

Theorem

$$\lim_{x \to a} f(x) = L \qquad \textit{if and only if} \qquad \lim_{x \to a^-} f(x) = L \textit{ and } \lim_{x \to a^+} f(x) = L$$

Instantaneous Rate of Change

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$$\lim_{\Delta t \to 0} \frac{\Delta h}{\Delta t} = \lim_{t \to 1} \frac{h(t) - h(1)}{t - 1}.$$

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Definition (Instantaneous Rate of Change)

The instantaneous rate of change of a function f at the input value x_1 is

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

provided the limit exists.

Instantaneous Rate of Change

An alternative, but equivalent definition for the instantaneous rate of change is

Definition (Instantaneous Rate of Change)

The instantaneous rate of change of a function f at the input value a is

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

provided the limit exists.

Think of $x_1 = a$, $x_2 = a + h$, then $\Delta x = h$.

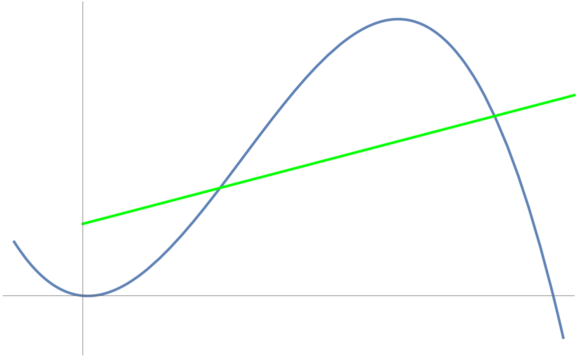
Instantaneous Rate of Change

Example

A rock is dropped from a bridge over a river. The distance, in meters, between the rock and the fiver t seconds after the rock is dropped is given by $s(t) = 48 - 4.9t^2$. Compute the speed of the rock after 2 seconds.

Slope of the Tangent Line

Recall that the average rate of change was the slope of a *secant line*. As we shrink the interval that the average rate of change is taken over, the slope of the secant lines approaches the slope of the tangent line.



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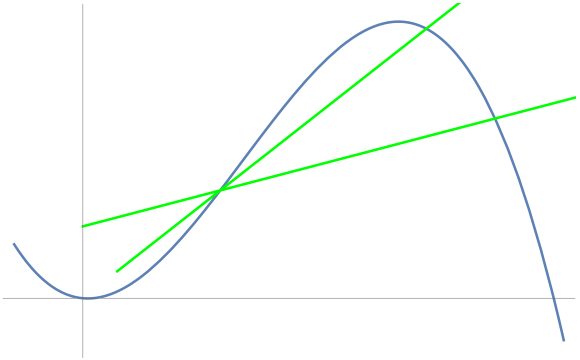

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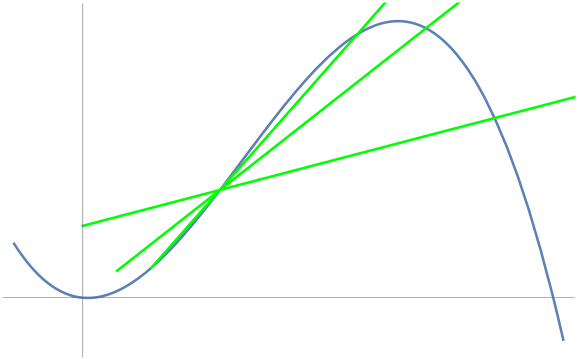

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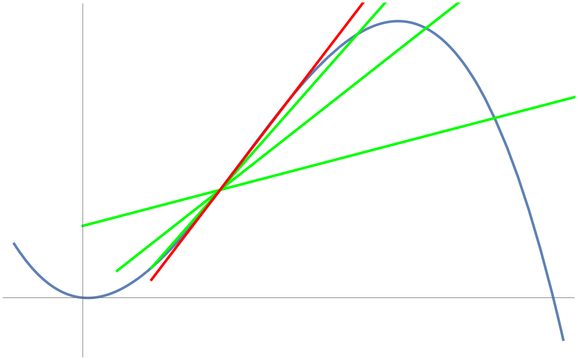

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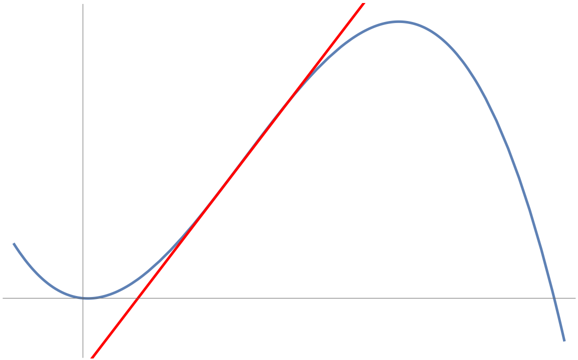

Slope of the Tangent Line

Definition (Slope of Tangent Line)

The tangent line to the curve $y = f(x)$ at the point $(x_1, f(x_1))$ is the line though this point with slope

$$m = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

provided the limit exists.

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The tangent line to the curve $y = f(x)$ at the point $(a, f(a))$ is the line though this point with slope

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

provided the limit exists.

Finding the Tangent Line

Example

Find the equation of the tangent line to the given function at the given point:

1. $y = 2x^2 + 1$ *at* $(3, 19)$

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1. $y = 2x^2 + 1$ *at* $(3, 19)$
2. $f(x) = 3x - x^2$ *at* $(1, 2)$

The Derivative

The slope, or instantaneous rate of change, of a function is usually referred to as *the derivative* of the function, denoted $f'(a)$.

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Definition (Derivative)

The derivative of the function f at the number a, denoted $f'(a)$, is defined by

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

if the limit exists.

Likewise, we could use the alternate form of the difference quotient to compute a derivative as well

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

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Compute the derivatives of the following functions at the given value:

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1. $g(t) = t^2 + 4t$, $t = 1$
2. $f(x) = \sqrt{2x}$, $x = 2$