The Derivative

Math 130 - Essentials of Calculus

4 October 2019

The Derivative - Revisited

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For a function $f(x)$, the derivative of f is the function $f'(x)$ defined by

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- The domain of f' consists of the values in the domain of $f(x)$ for which the limit above exists.
- The function $f(x)$ is said to be *differentiable* at $x = a$ if the derivative $f'(a)$ exists.

Comparing the Graphs of f and f'

Example

For the given function $f(x)$, (a) find $f'(x)$, (b) compare the graphs of f and f'.

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2. $f(x) = 2x^2 - 3$

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Theorem

If a function is differentiable at a number, then it is continuous there.

Alternative Notations

Recall that an alternative notation we had for a difference quotient for $y = f(x)$ was

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

In this notation, the derivative would be

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}.$$

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That is, we have:

$$f'(x) = \frac{dy}{dx} \qquad f'(a) = \left.\frac{dy}{dx}\right|_{x=a}$$

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Example

Compute the second derivative of $f(x) = 2x^2 - 3$ and compare the graph of f'' to the graph of f and f'.

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