Introduction to Integration

Math 130 - Essentials of Calculus

2 December 2019

The Fundamental Theorem of Calculus

Let's watch a video!

`https://youtu.be/rfG8ce4nNh0`

We will discuss some questions afterward.

Questions

1. How does the velocity graph relate to distance traveled?

Questions

1. How does the velocity graph relate to distance traveled?
2. How do we estimate the area under the curve?

Questions

1. How does the velocity graph relate to distance traveled?
2. How do we estimate the area under the curve?
3. How do we improve this estimate and ultimately get the area under the curve?

Questions

1. How does the velocity graph relate to distance traveled?
2. How do we estimate the area under the curve?
3. How do we improve this estimate and ultimately get the area under the curve?
4. What is the notation for the area under the curve?

Questions

1. How does the velocity graph relate to distance traveled?
2. How do we estimate the area under the curve?
3. How do we improve this estimate and ultimately get the area under the curve?
4. What is the notation for the area under the curve?
5. What is the notation for the distance traveled in T seconds? Draw a picture as well!

Questions

1. How does the velocity graph relate to distance traveled?
2. How do we estimate the area under the curve?
3. How do we improve this estimate and ultimately get the area under the curve?
4. What is the notation for the area under the curve?
5. What is the notation for the distance traveled in T seconds? Draw a picture as well!
6. How are antiderivatives useful in this problem?

Questions

1. How does the velocity graph relate to distance traveled?
2. How do we estimate the area under the curve?
3. How do we improve this estimate and ultimately get the area under the curve?
4. What is the notation for the area under the curve?
5. What is the notation for the distance traveled in T seconds? Draw a picture as well!
6. How are antiderivatives useful in this problem?
7. What is the Fundamental Theorem of Calculus?

Questions

1. How does the velocity graph relate to distance traveled?
2. How do we estimate the area under the curve?
3. How do we improve this estimate and ultimately get the area under the curve?
4. What is the notation for the area under the curve?
5. What is the notation for the distance traveled in T seconds? Draw a picture as well!
6. How are antiderivatives useful in this problem?
7. What is the Fundamental Theorem of Calculus?
8. What happens if the function we integrate takes on negative values? What does the integral compute here?

Integral Terminology

In the integral

$$\int_a^b f(x)\,dx$$

- $f(x)$ is called the **integrand**
- a and b are called the **limits of integration**
- specifically, a is the **lower limit** and b is the **upper limit**

Total Cost

Suppose we are given the marginal cost function for a good: $M(q)$. How could we use the marginal cost to find the total cost?

Total Cost

Suppose we are given the marginal cost function for a good: $M(q)$. How could we use the marginal cost to find the total cost?

If we wanted to, for example, compute the cost of producing 100 of the product, we would compute

$$\int_0^{100} M(q)\, dq.$$

Total Cost

Suppose we are given the marginal cost function for a good: $M(q)$. How could we use the marginal cost to find the total cost?

If we wanted to, for example, compute the cost of producing 100 of the product, we would compute

$$\int_0^{100} M(q)\, dq.$$

This relationship works since the marginal cost function is just the derivative of the total cost function.

Total Cost

Suppose we are given the marginal cost function for a good: $M(q)$. How could we use the marginal cost to find the total cost?

If we wanted to, for example, compute the cost of producing 100 of the product, we would compute

$$\int_0^{100} M(q)\, dq.$$

This relationship works since the marginal cost function is just the derivative of the total cost function. In other words, total cost is an antiderivative of marginal cost!

Total Cost

Suppose we are given the marginal cost function for a good: $M(q)$. How could we use the marginal cost to find the total cost?

If we wanted to, for example, compute the cost of producing 100 of the product, we would compute

$$\int_0^{100} M(q)\, dq.$$

This relationship works since the marginal cost function is just the derivative of the total cost function. In other words, total cost is an antiderivative of marginal cost!

Using integration, we are even able to see how production costs would increase if we wanted to increase the amount of product produced. For example, if we wanted to see the additional costs involved in raising production from 200 to 400 units, we would just compute

$$\int_{200}^{400} M(q)\, dq.$$

Examples

Compute the following integrals

1. $\int_0^4 (6x - 5)dx$

Examples

Compute the following integrals

1. $\displaystyle\int_0^4 (6x - 5)dx$
2. $\displaystyle\int_{-1}^3 x^5 \, dx$

Examples

Compute the following integrals

1. $\displaystyle\int_0^4 (6x-5)dx$
2. $\displaystyle\int_{-1}^3 x^5\, dx$
3. $\displaystyle\int_3^5 (t^2+1)dx$

Examples

Compute the following integrals

1. $\int_0^4 (6x - 5)dx$
2. $\int_{-1}^3 x^5 \, dx$
3. $\int_3^5 (t^2 + 1)dx$
4. $\int_1^3 (1_2 x - 4x^3)dx$

Examples

Compute the following integrals

1. $\int_0^4 (6x - 5)dx$
2. $\int_{-1}^3 x^5 \, dx$
3. $\int_3^5 (t^2 + 1)dx$
4. $\int_1^3 (1_2 x - 4x^3)dx$
5. $\int_1^9 4\sqrt{z} \, dz$

Examples

Compute the following integrals

1. $\displaystyle\int_0^4 (6x - 5)dx$
2. $\displaystyle\int_{-1}^3 x^5 \, dx$
3. $\displaystyle\int_3^5 (t^2 + 1)dx$
4. $\displaystyle\int_1^3 (1_2 x - 4x^3)dx$
5. $\displaystyle\int_1^9 4\sqrt{z} \, dz$
6. $\displaystyle\int_{-1}^0 (2x - e^x)dx$