Computing Derivatives

Math 130 - Essentials of Calculus

1 March 2021

Constant Multiple Rule

Theorem

Let c be a real number,

$$\frac{d}{dx}[cf(x)] = cf'(x).$$

Constant Multiple Rule

Theorem

Let c be a real number,

$$\frac{d}{dx}[cf(x)] = cf'(x).$$

Example

Compute the derivative of the following functions:

1. $f(x) = 9x^4$

Constant Multiple Rule

Theorem

Let c be a real number,

$$\frac{d}{dx}[cf(x)] = cf'(x).$$

Example

Compute the derivative of the following functions:

1. $f(x) = 9x^4$
2. $g(q) = 3\sqrt{q}$

Constant Multiple Rule

Theorem

Let c be a real number,

$$\frac{d}{dx}[cf(x)] = cf'(x).$$

Example

Compute the derivative of the following functions:

1. $f(x) = 9x^4$
2. $g(q) = 3\sqrt{q}$
3. $f(x) = 9\sqrt[3]{x}$
4. $w(z) = \frac{4}{z^3}$

Derivatives of Sums and Differences

Theorem

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

$$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$

Derivatives of Sums and Differences

Theorem

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$
$$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$

Example

Compute the derivative of the following functions:

1. $f(x) = \frac{7}{4}x^2 - 3x + 12$

Derivatives of Sums and Differences

Theorem

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

$$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$

Example

Compute the derivative of the following functions:

1. $f(x) = \frac{7}{4}x^2 - 3x + 12$
2. $g(x) = x^{5/3} - x^{2/3}$

Derivatives of Sums and Differences

Theorem

$$\frac{d}{dx}[f(x)+g(x)] = f'(x)+g'(x)$$

$$\frac{d}{dx}[f(x)-g(x)] = f'(x)-g'(x)$$

Example

Compute the derivative of the following functions:

1. $f(x) = \frac{7}{4}x^2 - 3x + 12$
2. $g(x) = x^{5/3} - x^{2/3}$
3. $f(x) = x^5 - 2x^3 + x - 1$
4. $w(z) = 2x - 5x^{3/4}$

Exponential Functions

An *exponential function* is a function of the form

$$f(x) = b^x$$

where b is a positive constant called the *base*. The domain of an exponential function is $\mathbb{R} = (-\infty, \infty)$. A slightly more general exponential function is

$$f(x) = C \cdot b^x$$

where C represents the initial value of f (because $f(0) = C$).

Example

Graph the following exponential functions:

1. $f(x) = 2^x$
2. $f(x) = \left(\frac{1}{2}\right)^x$

The Natural Number e

We want to take the derivative of exponential functions $f(x) = b^x$. Let's start by computing the derivative at $x = 0$:

The Natural Number e

We want to take the derivative of exponential functions $f(x) = b^x$. Let's start by computing the derivative at $x = 0$:

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

The Natural Number e

We want to take the derivative of exponential functions $f(x) = b^x$. Let's start by computing the derivative at $x = 0$:

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{b^h - 1}{h}.$$

The Natural Number e

We want to take the derivative of exponential functions $f(x) = b^x$. Let's start by computing the derivative at $x = 0$:

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{b^h - 1}{h}.$$

If we use $b = 2$, we'll find that $f'(0) < 1$ and if we use $b = 3$, we'll find that $f'(0) > 1$.

The Natural Number *e*

We want to take the derivative of exponential functions $f(x) = b^x$. Let's start by computing the derivative at $x = 0$:

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{b^h - 1}{h}.$$

If we use $b = 2$, we'll find that $f'(0) < 1$ and if we use $b = 3$, we'll find that $f'(0) > 1$. It them seems reasonable to guess that there's a value of b that will make $f'(0) = 1$.

THE NATURAL NUMBER e

We want to take the derivative of exponential functions $f(x) = b^x$. Let's start by computing the derivative at $x = 0$:

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{b^h - 1}{h}.$$

If we use $b = 2$, we'll find that $f'(0) < 1$ and if we use $b = 3$, we'll find that $f'(0) > 1$. It them seems reasonable to guess that there's a value of b that will make $f'(0) = 1$. There, in fact, is, and that number is called the *natural number e* (which has value approximately 2.71828).

The Natural Number e

We want to take the derivative of exponential functions $f(x) = b^x$. Let's start by computing the derivative at $x = 0$:

$$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{b^h - 1}{h}.$$

If we use $b = 2$, we'll find that $f'(0) < 1$ and if we use $b = 3$, we'll find that $f'(0) > 1$. It them seems reasonable to guess that there's a value of b that will make $f'(0) = 1$. There, in fact, is, and that number is called the *natural number e* (which has value approximately 2.71828). The function $f(x) = e^x$ turns out to have some really nice properties in terms of calculus.

The Natural Number e

We want to take the derivative of exponential functions $f(x) = b^x$. Let's start by computing the derivative at $x = 0$:

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{b^h - 1}{h}.$$

If we use $b = 2$, we'll find that $f'(0) < 1$ and if we use $b = 3$, we'll find that $f'(0) > 1$. It them seems reasonable to guess that there's a value of b that will make $f'(0) = 1$. There, in fact, is, and that number is called the *natural number e* (which has value approximately 2.71828). The function $f(x) = e^x$ turns out to have some really nice properties in terms of calculus. So, by the way we chose e, the following limit is true:

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1.$$

The Natural Number e

We want to take the derivative of exponential functions $f(x) = b^x$. Let's start by computing the derivative at $x = 0$:

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{b^h - 1}{h}.$$

If we use $b = 2$, we'll find that $f'(0) < 1$ and if we use $b = 3$, we'll find that $f'(0) > 1$. It them seems reasonable to guess that there's a value of b that will make $f'(0) = 1$. There, in fact, is, and that number is called the *natural number e* (which has value approximately 2.71828). The function $f(x) = e^x$ turns out to have some really nice properties in terms of calculus. So, by the way we chose e, the following limit is true:

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1.$$

Derivative of the Exponential Function

If $f(x) = e^x$, then we have

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivative of the Exponential Function

If $f(x) = e^x$, then we have

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$$

Derivative of the Exponential Function

If $f(x) = e^x$, then we have

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} \\ &= \lim_{h \to 0} \frac{e^x(e^h - 1)}{h} \end{aligned}$$

Derivative of the Exponential Function

If $f(x) = e^x$, then we have

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} \\ &= \lim_{h\to 0} \frac{e^x(e^h - 1)}{h} = e^x \lim_{h\to 0} \frac{e^h - 1}{h} \end{aligned}$$

Derivative of the Exponential Function

If $f(x) = e^x$, then we have

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} \\ &= \lim_{h\to 0} \frac{e^x(e^h - 1)}{h} = e^x \lim_{h\to 0} \frac{e^h - 1}{h} = e^x \cdot 1 \end{aligned}$$

Derivative of the Exponential Function

If $f(x) = e^x$, then we have

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} \\ &= \lim_{h\to 0} \frac{e^x(e^h - 1)}{h} = e^x \lim_{h\to 0} \frac{e^h - 1}{h} = e^x \cdot 1 = e^x \end{aligned}$$

Derivative of the Exponential Function

If $f(x) = e^x$, then we have

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{e^{x+h}-e^x}{h} \\ &= \lim_{h\to 0} \frac{e^x(e^h-1)}{h} = e^x \lim_{h\to 0} \frac{e^h-1}{h} = e^x \cdot 1 = e^x \end{aligned}$$

Property

$$\frac{d}{dx}[e^x] = e^x$$

Derivative of the Exponential Function

If $f(x) = e^x$, then we have

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{e^{x+h}-e^x}{h} \\ &= \lim_{h\to 0} \frac{e^x(e^h-1)}{h} = e^x \lim_{h\to 0} \frac{e^h-1}{h} = e^x \cdot 1 = e^x \end{aligned}$$

Property

$$\frac{d}{dx}[e^x] = e^x$$

We will talk about the derivative of the more general exponential function, $f(x) = b^x$, soon, but we need a bit more derivative tech first.

Computing Second Derivatives

Example

Compute the second derivative of the following functions:

1. $f(t) = \frac{1}{6}t^6 - 3t^4 + t$

Computing Second Derivatives

Example

Compute the second derivative of the following functions:

1. $f(t) = \frac{1}{6}t^6 - 3t^4 + t$
2. $f(x) = x^3 - 4x + 6$
3. $g(t) = (t-2)(2t+3)$

Basic Physics

If $s(t)$ is a position function, then the velocity is given by $v(t) = s'(t)$ and acceleration is given by $a(t) = v'(t) = s''(t)$.

Basic Physics

If $s(t)$ is a position function, then the velocity is given by $v(t) = s'(t)$ and acceleration is given by $a(t) = v'(t) = s''(t)$.

Example

The equation of motion of a moving object is $s(t) = t^4 - 8t^2 + 4$*, where* s *measures in meters and* t *measures in seconds.*

1. *Find the velocity and acceleration functions for the object.*

Basic Physics

If $s(t)$ is a position function, then the velocity is given by $v(t) = s'(t)$ and acceleration is given by $a(t) = v'(t) = s''(t)$.

Example

The equation of motion of a moving object is $s(t) = t^4 - 8t^2 + 4$, where s measures in meters and t measures in seconds.

1. *Find the velocity and acceleration functions for the object.*
2. *At what times is the object at rest (zero velocity)?*

Basic Physics

If $s(t)$ is a position function, then the velocity is given by $v(t) = s'(t)$ and acceleration is given by $a(t) = v'(t) = s''(t)$.

Example

The equation of motion of a moving object is $s(t) = t^4 - 8t^2 + 4$, *where s measures in meters and t measures in seconds.*

1. *Find the velocity and acceleration functions for the object.*
2. *At what times is the object at rest (zero velocity)?*
3. *At what times does the object change direction?*

Tangent Lines

Example

Find the tangent line to the given function at the given point.

1. $f(x) = e^x - x^3$ at $(0, 0)$

Tangent Lines

Example

Find the tangent line to the given function at the given point.

1. $f(x) = e^x - x^3$ at $(0, 0)$
2. $f(x) = x + \sqrt{x}$ at $(4, 6)$