Maximum and Minimum Values

Math 130 - Essentials of Calculus

31 March 2021

Definitions - Review

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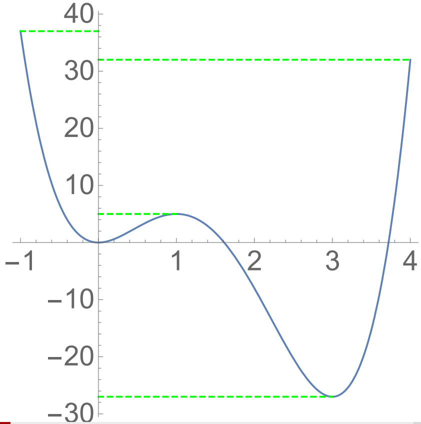
Extrema of a Function

Consider the function $f(x) = 3x^4 - 16x^3 + 18x^2$ on the domain $-1 \leq x \leq 4$. Where are the absolute maximum and absolute minimum values, and what are they? Are there any local minimum and local maximum values?



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2. *Considered on the domain* $(-\infty, \infty)$, $f(x) = x^3$ *has no extrema at all!*

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It turns out that if you consider a continuous function on a closed interval, of the form $[a, b]$, you're guaranteed an absolute maximum and minimum.

Theorem (The Extreme Value Theorem)

If f *is continuous on a closed interval, then it always attains an absolute maximum value and an absolute minimum value on that interval.*

Locating Extreme Values

Observing some of the pictures we've had so far, the following theorem is apparent:

Theorem (Fermat's Theorem)

If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

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It's possible that a function could have a local extrema at a place where $f'(c) \neq 0$, for example, consider $f(x) = |x|$. It turns out that what we're really looking for are *critical numbers*.

Definition (Critical Number)

A critical number of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

Critical Numbers - Examples

Example

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2. $f(x) = \sqrt[3]{x}$
3. $g(x) = x^3 + 3x^2 - 24$

The Closed Interval Method

Theorem

To find the absolute maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. *Find the critical numbers of f in the interval (a, b) and compute the values of f at these numbers.*
2. *Find the values of f at the endpoints of the interval.*
3. *The largest of the output values from steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.*

Finding Extrema

Example

Find the absolute maximum and absolute minimum values of f on the given interval.

1. $f(x) = x^3 - 3x + 1$, $[0, 3]$
2. $f(x) = x^3 - 6x^2 + 9x + 2$, $[-1, 4]$