Finding Local Extrema - The First and Second Derivative Tests

Math 130 - Essentials of Calculus

2 April 2021

Increasing/Decreasing

Theorem

1. *If* $f'(x) > 0$ *on an interval, then* $f(x)$ *is increasing on that interval.*
2. *If* $f'(x) < 0$ *on an interval, then* $f(x)$ *is decreasing on that interval.*

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Example

Find the intervals on which the given function is increasing and decreasing:

1. $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

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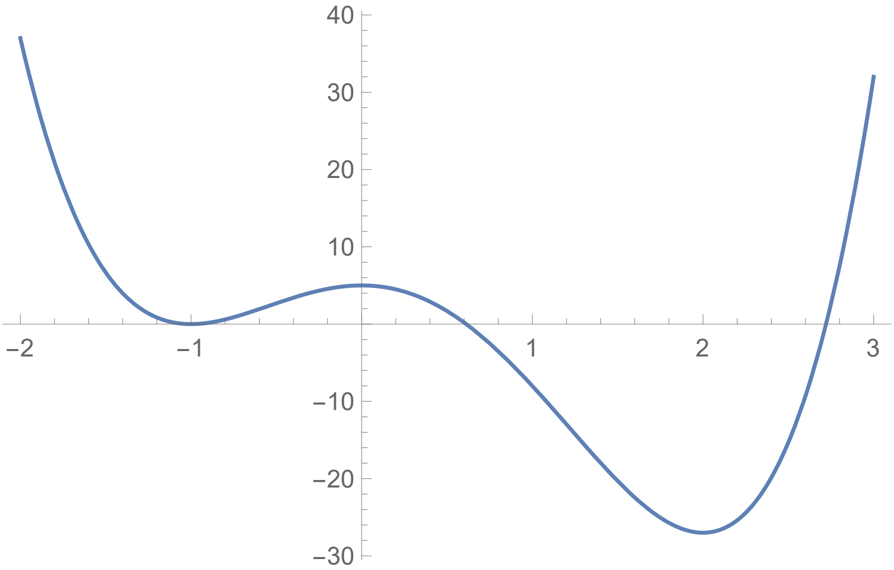
2. $f(x) = 2x^3 - 3x^2 - 12x$


40
30
20
10
−10
−20
−30
−2
−1
1
2
3

The First Derivative Test

Theorem (The First Derivative Test)

Suppose that c is a critical number of a continuous function f.

1. *If f' changes from positive to negative at c, then f has a local maximum at c.*
2. *If f' changes from negative to positive at c, then f has a local minimum at c.*
3. *If f' does not change sign at c (for example, if f' is positive on both sides of c or negative on both sides), then f has no local maximum or minimum at c.*

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Review - Concavity

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Theorem

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Suppose f'' is continuous near c and that $f'(c) = 0$.

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