Concavity and Optimization Start

Math 130 - Essentials of Calculus

6 April 2021

Review - Concavity

Definition

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Theorem

1. *If $f''(x) > 0$ on an interval, then $f(x)$ is concave upward on that interval.*
2. *If $f''(x) < 0$ on an interval, then $f(x)$ is concave downward on that interval.*

The Second Derivative Test

Theorem (The Second Derivative Test)

Suppose f'' is continuous near c and that $f'(c) = 0$.

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Example

Find the local maximum and minimum values of $y = x^4 - 4x^3$.

Concavity Example

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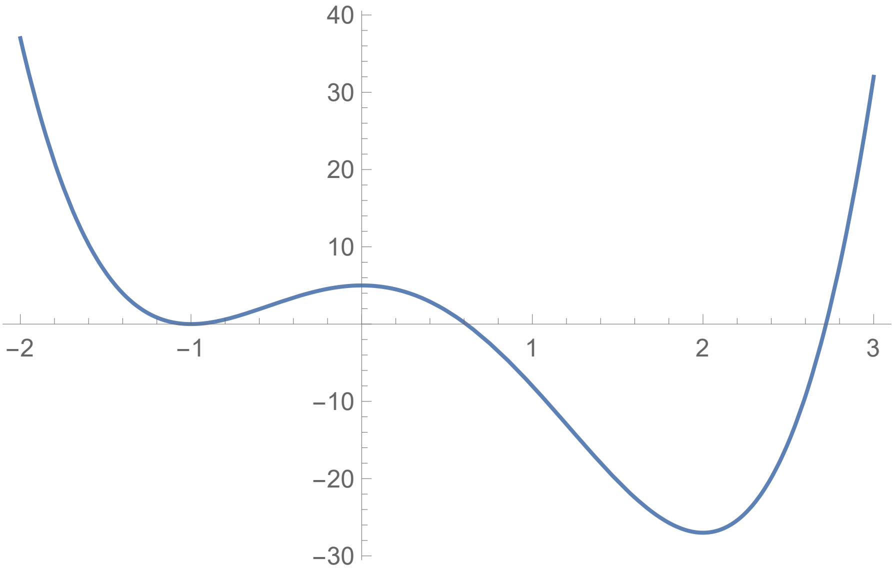
40
30
20
10
−10
−20
−30
−2
−1
1
2
3

Starting Example

Example

A farmer has 2400ft of fencing and wants to fence off a rectangular field that borders a straight river. She needs no fence along the river. What are the dimensions of the field that has the largest area?

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4. *Determine the feasible domain of the primary equation. That is, determine the values for which the stated problem makes sense.*
5. *Determine the desired maximum or minimum value using calculus.*

Now You Try It!

Example

Find the dimensions of a rectangle with perimeter $100m$ *whose area is as large as possible.*

Example without a Feasible Domain

Example

Find the dimensions of a rectangle with area $1000m^2$ *whose perimeter is as small as possible.*

Now You Try It!

Example

A box with a square base and open top must have a volume of $32,000cm^3$. *Find the dimensions of the box that minimize the amount of material used.*

Additional Examples

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1. *Find two numbers whose difference is* 100 *and whose product is a maximum.*
2. *Find two positive numbers whose product is* 100 *and whose sum is a minimum.*
3. *Find a positive number such that the sum of the number and its reciprocal is as small as possible.*