Optimization and Elasticity

Math 130 - Essentials of Calculus

12 April 2021

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$$c'(q) = \frac{d}{dq}\frac{C(q)}{q} = \frac{C'(q)q - C(q)}{q^2}.$$

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$$C'(q) = \frac{C(q)}{q}.$$

Maximizing Profit

Recall that the revenue function R gives the total revenue collected after q units are sold. Combining this with the cost function, we can get the profit function

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$P'(q) = 0$ when $R'(q) = C'(q)$, as we had previously discovered! To ensure any solutions are a maximum, we apply the Second Derivative Test which says we want $P''(q) < 0$. Since $P''(q) = R''(q) - C''(q)$, this means that $R''(q) < C''(q)$.

Elasticity

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What we want to discuss now is how strongly a change in price influences a change in demand. For example, a 20% change in price may or may not have a drastic effect on demand. As an example, if restaurants suddenly raised their prices by 20%, it's likely many customers will choose to stay home instead. However, if gasoline prices were to go up by 20%, demand would be affected, but not by that much since people still need it.

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Definition

Elasticity The ***elasticity of demand*** *E for a product whose demand q corresponds to the price $p = D(q)$ is given by*

$$E(q) = -\frac{p/q}{dp/dq} = -\frac{D(q)}{qD'(q)}.$$

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Example

A tool company estimates that the monthly demand q for their power drill is related to the price p for each drill by $p = 185 - 0.06q$. Compute the elasticity of demand for drill prices of \$50 and \$95.

Now You Try It!

Example

The demand function for a particular pair of sunglasses is

$$p = 155 - 0.035q.$$

1. *If the sunglasses are priced as \$65, how many pairs can be sold?*
2. *Compute the elasticity of demand when the sunglasses are priced at \$65 and interpret your result. At this price, is the demand elastic or inelastic?*

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unit elastic	$E(q) = 1$	at maximum

Elasticity and Maximizing Revenue

Example

The demand function for a manufacturer's product is $D(q) = 75e^{-0.05q}$. Write a formula for the elasticity of demand E and determine the price per unit that maximizes revenue.

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If the demand function for a particular purse is $p = 150 - 4\sqrt{q}$, use elasticity to find the price and corresponding quantity that maximize revenue.