Integration by Parts

Math 130 - Essentials of Calculus

5 May 2021

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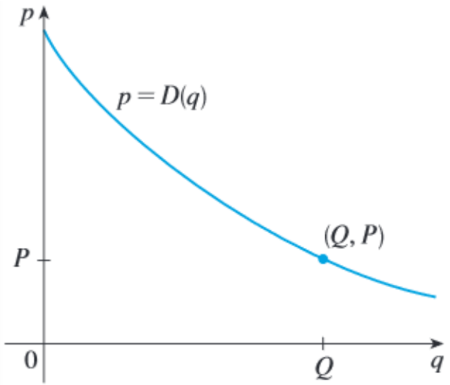

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What we'd like to find is the *total consumer surplus*, the sum over all purchasers of the product. This represents a sort of ”total savings” to society when the product is sold at a certain price.
To compute this, we need to find a certain area under the demand curve, above the current price of the item.

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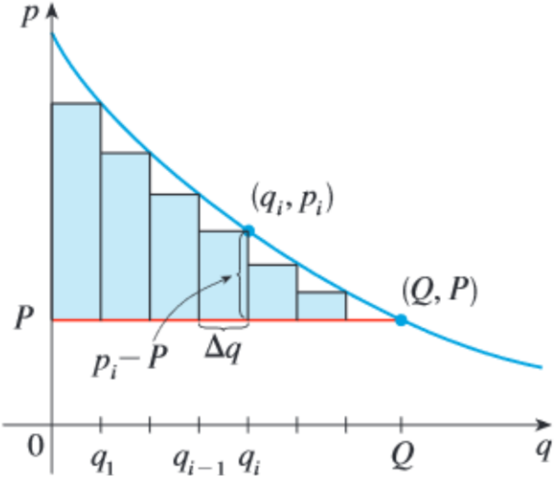

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Definition (Consumer Surplus)

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1. *The demand for a product, in dollars, is $p = 1200 - 0.2q - 0.0001q^2$. Find the consumer surplus when the sales level is 500.*
2. *The demand function for a particular vacation package is $D(q) = 2000 - 46\sqrt{q}$. Find the consumer surplus when the sales level for the package is 800.*

Producer Surplus

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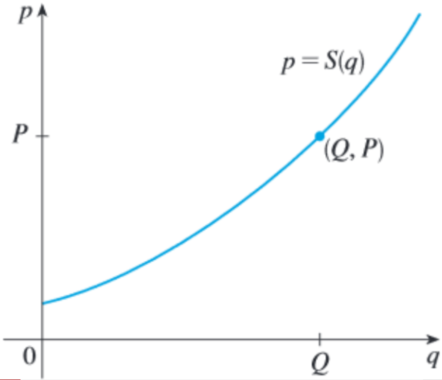

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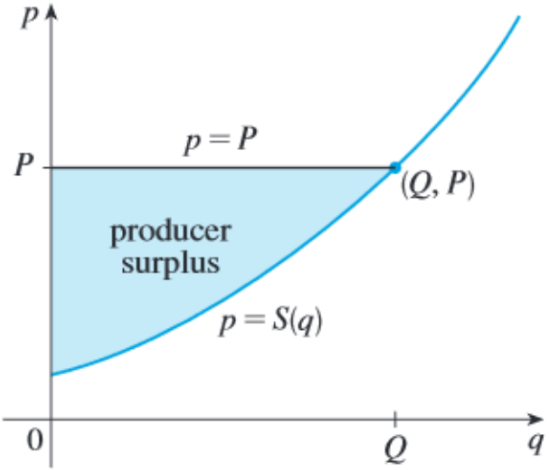
If a producer can sell a product for more than their minimum acceptable price, this benefits the producer. The amount above the minimum price is called *producer surplus*. As with consumer surplus, we would like to find the *total producer surplus*. If we let $P = S(Q)$ be the current market price and quantity sold. By using a similar process to how we found total consumer surplus, we find the total producer surplus to be the area below market price and above the supply curve.

Producer Surplus



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1. *An electronics manufacturer estimates that the supply function for it's digital clocks is $S(q) = 5.4 + 0.001q^{1.2}$ dollars. Find the producer surplus when the number of clocks sold is 2000.*
2. *If a supply curve is modeled by the equation $p = 200 + 0.2q^{3/2}$, find the producer surplus when the selling price is \$400.*

Total Surplus

It is assumed that in a competitive market, the price of a product is naturally driven to a value where demand from consumers equals the quantity producers are willing and able to sell. This point is when the market is said to be in *equilibrium*. Graphically, this is where the supply and demand curves cross.

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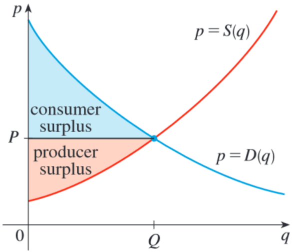
The sum of consumer and producer surplus is called *total surplus*. This is the area between the supply and demand curves for $0 \leq q \leq Q$. This quantity is maximized when Q is at equilibrium.



Maximizing Total Surplus

Example

The demand function for metal thermoses produced by a manufacturer is $p = D(q) = 14e^{-0.15q}$ and the supply function is $p = S(q) = 2e^{0.12q}$ where q is measured in thousands. What price should the thermoses sell for in order to maximize the total surplus? What is the total surplus?