

$y = \ln x$. What is $y' =$?

If $y = \ln x$, then $e^y = x$.

$$\frac{d}{dx}(e^y) = \frac{d}{dx}(x)$$

$$e^y \cdot y' = 1$$

$$\rightarrow y' = \frac{1}{e^y} = \frac{1}{e^{\ln x}} = \frac{1}{x}$$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

$$y = \log_b x$$

↕

$$b^y = x$$

$$\frac{d}{dx}[e^{f(x)}]$$
$$= e^{f(x)} \cdot f'(x)$$

What is $\frac{d}{dx}[\log_b x]$?

$$y = \log_b x = \frac{\ln x}{\ln b}$$

$$\log_a x = \frac{\log_b x}{\log_b a} = \frac{\ln x}{\ln a}$$

Change of Base

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{\ln x}{\ln b}\right] = \frac{1}{\ln b} \cdot \frac{d}{dx}$$

