

**Question.** Consider the function

$$f(x, y) = \begin{cases} \frac{x^3y - xy^3}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

- (a) Find  $f_x(x, y)$  and  $f_y(x, y)$  when  $(x, y) \neq (0, 0)$ .
- (b) Find  $f_x(0, 0)$  and  $f_y(0, 0)$ . You will need to use the definitions of partial derivatives at a point to do this.
- (c) Find  $f_{xy}(0, 0)$  and  $f_{yx}(0, 0)$ . You will again need to use the definition of partial derivatives at a point.
- (d) Reconcile this with Clairaut's theorem.