

Math 307 - Differential Equations - Spring 2017

Quiz 8
April 6, 2017

Name: Solution

Problem 1. Solve the differential equation

$$(2x + 1)y'' - 2y' - (2x + 3)y = (2x + 1)^2 e^{-x}$$

given that $y_1 = e^{-x}$ and $y_2 = xe^x$ are homogeneous solutions of the equation.

Variation of Parameters

$$\textcircled{1} \quad u_1' e^{-x} + u_2' x e^x = 0$$

$$\textcircled{2} \quad -u_1' e^{-x} + u_2' (e^x + x e^x) = \frac{(2x+1)^2 e^{-x}}{2x+1} = (2x+1)e^{-x}$$

$$\textcircled{1} + \textcircled{2} : u_2' (2x e^x + e^x) = (2x+1)e^{-x}$$

$$\Rightarrow u_2' \cancel{(2x+1)} e^x = \cancel{(2x+1)} e^{-x} \Rightarrow u_2' = e^{-2x} \Rightarrow u_2 = \frac{-1}{2} e^{-2x} + C_2$$

$$\textcircled{1} \Rightarrow u_1' e^{-x} = -u_2' x e^x = -(e^{-2x}) x e^x = -x e^{-x}$$

$$\Rightarrow u_1' = -x \Rightarrow u_1 = -\frac{1}{2} x^2 + C_1$$

$$\Rightarrow Y_G = u_1 y_1 + u_2 y_2 = \left(-\frac{1}{2} x^2 + C_1\right) e^{-x} + \left(\frac{-1}{2} e^{-2x} + C_2\right) x e^x$$

$$\left[\Rightarrow Y_G = \underbrace{C_1 e^{-x} + C_2 x e^x}_{\text{homogeneous}} + \underbrace{\frac{-1}{2} e^{-x} (x^2 + x)}_{\text{particular}} \right]$$