HW #4

6.3 # 2, 4, 6, 8, 16, 19abcf, 36

6.4 # 2, 6, 10, 15e, 22 (try 21)

6.3) ② Clearly the columns are orthogonal each with norm 1.

$$A=\begin{pmatrix} 3/5 & 0 & 4/5 \\ -4/5 & 0 & 3/5 \\ 0 & 1 & 0 \end{pmatrix} \Rightarrow A^{-1}=\begin{pmatrix} 3/5 & -4/5 & 0 \\ 0 & 0 & 1 \\ 4/5 & 3/5 & 0 \end{pmatrix}$$

④ Clearly the columns are orthogonal with length 1.

$$A=\frac{1}{2}\begin{pmatrix} 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix} \Rightarrow A^{-1}=A \text{ since } A \text{ is symmetric}$$

⑥

$$A=\begin{array}{c} \begin{pmatrix} 3 & 0 & 8 \\ -4 & 0 & 6 \\ 0 & 1 & 0 \end{pmatrix} \\ \begin{matrix} v_1 & v_2 & v_3 \end{matrix} \end{array} \qquad \begin{aligned} \|v_1\| &= 5 \\ \|v_2\| &= 1 \\ \|v_3\| &= 10 \end{aligned}$$

So let

$$D=\begin{pmatrix} 1/5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/10 \end{pmatrix}.$$

Then

$$AD=\begin{pmatrix} 3/5 & 0 & 4/5 \\ -4/5 & 0 & 3/5 \\ 0 & 1 & 0 \end{pmatrix} \text{ is orthogonal.}$$

So

$$A^{-1}=D^2A^T=\begin{pmatrix} 1/25 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/100 \end{pmatrix}\begin{pmatrix} 3 & -4 & 0 \\ 0 & 0 & 1 \\ 8 & 6 & 0 \end{pmatrix}=\begin{pmatrix} 3/25 & -4/25 & 0 \\ 0 & 0 & 1 \\ 2/25 & 3/50 & 0 \end{pmatrix}$$

(8)

$$A = \begin{pmatrix} 2 & -1 & 3 & 1 \\ -2 & 1 & 3 & 1 \\ 2 & 1 & -3 & 1 \\ 2 & 1 & 3 & -1 \end{pmatrix} \qquad \begin{aligned} \|v_1\| &= 4 \\ \|v_2\| &= 2 \\ \|v_3\| &= 6 \\ \|v_4\| &= 2 \end{aligned}$$

$$\begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array}$$

So let

$$D = \begin{pmatrix} 1/4 & & & \\ & 1/2 & & 0 \\ & 0 & 1/6 & \\ & & & 1/2 \end{pmatrix}$$

Then

$$AD = \frac{1}{2}\begin{pmatrix} 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix}$$

is orthonormal.

So:

$$A^{-1} = D^2 A^T = \frac{1}{4}\begin{pmatrix} 1/2 & -1/2 & 1/2 & 1/2 \\ -1 & 1 & 1 & 1 \\ 1/3 & 1/3 & -1/3 & 1/3 \\ 1 & 1 & 1 & -1 \end{pmatrix}$$

(16)

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

e.vals: $\lambda_1 = -1,\ \lambda_2 = 0,\ \lambda_3 = 3$

e.vecs: $v_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix},\ v_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix},\ v_3 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

Normalize the e.vecs and make C:

$$C = \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \\ 0 & -1/\sqrt{3} & 2/\sqrt{6} \\ 1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \end{pmatrix} \quad \text{So} \quad D = \begin{pmatrix} -1 & & 0 \\ & 0 & \\ 0 & & 3 \end{pmatrix}$$

(19) (a) False. Need to be orthonormal

(b) True. The matrix is invertible.

(c) True. Let $B = A^T$ then $B^T B = (A^T)^T A^T$

$= AA^T = (AA^T)^T = (I)^T = I$

(f) True. $A^T A = I$ & $B^T B = I$

$(AB)^T AB = B^T (A^T A) B = B^T I B = B^T B = I$.

(36) $T((x,y)) = \left(\frac{x}{2} - \frac{\sqrt{3}}{2}y, -\frac{\sqrt{3}}{2}x + \frac{y}{2}\right) = \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$\therefore$ T is not orthogonal since the columns are not perpendicular.

6.4) (2) Let

$$A = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \text{ then } A^T = (1 \ -1 \ 2)$$

So

$$P = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \left[(1 \ -1 \ 2) \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \right]^{-1} (1 \ -1 \ 2)$$

$$= \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} (6)^{-1} (1 \ -1 \ 2) = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \left(\frac{1}{6}\right) (1 \ -1 \ 2)$$

$$= \frac{1}{6} \begin{pmatrix} 1 & -1 & 2 \\ -1 & 1 & -2 \\ 2 & -2 & 4 \end{pmatrix}$$

Thus

$$\text{proj}_W (1,3,4) = \frac{1}{6} \begin{pmatrix} 1 & -1 & 2 \\ -1 & 1 & -2 \\ 2 & -2 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

(6) A basis for the plane is:

$$\left\{ \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} \right\}$$

Let

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -3 & -2 \end{pmatrix} \Rightarrow A^T = \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & -2 \end{pmatrix}$$

So

$$P = A(A^TA)^{-1}A^T = \frac{1}{14}\begin{pmatrix} 5 & -6 & -3 \\ -6 & 10 & -2 \\ -3 & -2 & 13 \end{pmatrix}$$

Then

$$\text{proj}_{\text{plane}}(4,2,-1) = \frac{1}{14}\begin{pmatrix} 5 & -6 & -3 \\ -6 & 10 & -2 \\ -3 & -2 & 13 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 11/14 \\ -1/7 \\ -29/14 \end{pmatrix}$$

(10)

$$W = sp\{e_1, e_3\}$$

So

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \Rightarrow A^T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

So

$$P = \begin{pmatrix} 1 & & & \\ & 0 & & 0 \\ & 0 & 1 & \\ & & & 0 \end{pmatrix}$$

15e False. Thm 6.11.

21 Suppose P^{-1} exists. Then since $P^2 = P$:

$$P^{-1}(P^2) = P^{-1}(P) = I$$

But $P^{-1}(P^2) = P$.

So $P = I$

22

$$W = sp\left\{\begin{pmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{pmatrix}, \begin{pmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix}\right\}$$

Let

$$A = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{3} \\ 0 & -1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{3} \end{pmatrix} \Rightarrow A^T = \begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}$$

Then

$$P = AA^T = \begin{pmatrix} 5/6 & -1/3 & -1/6 \\ -1/3 & 1/3 & -1/3 \\ -1/6 & -1/3 & 5/6 \end{pmatrix}$$