HW #5

6.5 #1 (using table $\begin{array}{c|c|c|c|c} W & 1 & 2 & 4 & 6 \\ \hline L & 3 & 4 & 6 & 8 \end{array}$), 6, 8, 13

7.1 #3, 4, 8, 12, 18

6.5) ①

weight (a_i)	1	2	4	6
length (b_i)	3	4	6	8

ⓐ

$$\begin{pmatrix}3\\4\\6\\8\end{pmatrix} = \underset{\overset{\shortparallel}{A}}{\begin{pmatrix}1&1\\1&2\\1&4\\1&6\end{pmatrix}}\begin{pmatrix}l\\k\end{pmatrix}$$

1
4
16
36

$$A^TA = \begin{pmatrix}1&1&1&1\\1&2&4&6\end{pmatrix}\begin{pmatrix}1&1\\1&2\\1&4\\1&6\end{pmatrix} = \begin{pmatrix}4&13\\13&57\end{pmatrix}$$

2
57
4
228 - 169

13
13
39
130
169

$$\boxed{(A^TA)^{-1} = \frac{1}{59}\begin{pmatrix}57&-13\\-13&4\end{pmatrix}}$$

59

$$\begin{pmatrix}l\\k\end{pmatrix} = \frac{1}{59}\begin{pmatrix}57&-13\\-13&4\end{pmatrix}\begin{pmatrix}1&1&1&1\\1&2&4&6\end{pmatrix}\begin{pmatrix}3\\4\\6\\8\end{pmatrix}$$

2
3
8
24
48
83

$$= \frac{1}{59}\begin{pmatrix}57&-13\\-13&4\end{pmatrix}\begin{pmatrix}21\\83\end{pmatrix} = \frac{1}{59}\begin{pmatrix}1197-1079\\-273+332\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix}$$

The line of best fit is
$f(x) = 2 + x$

ⓑ $f(5) = 7.$

⑥

a_i	1	2	3	4
b_i	1	4	6	9

$$\begin{pmatrix}1\\4\\6\\9\end{pmatrix}=\underset{\overset{\shortparallel}{A}}{\begin{pmatrix}1&1\\1&2\\1&3\\1&4\end{pmatrix}}\begin{pmatrix}r_0\\r_1\end{pmatrix}$$

1
4
9
16

$$A^TA=\begin{pmatrix}1&1&1&1\\1&2&3&4\end{pmatrix}\begin{pmatrix}1&1\\1&2\\1&3\\1&4\end{pmatrix}=\begin{pmatrix}4&10\\10&30\end{pmatrix}$$

$$(A^TA)^{-1}=\frac{1}{20}\begin{pmatrix}30&-10\\-10&4\end{pmatrix}$$

1 > 9
8
18 — 27
36 — 63

$$\begin{pmatrix}r_0\\r_1\end{pmatrix}=\frac{1}{20}\begin{pmatrix}30&-10\\-10&4\end{pmatrix}\begin{pmatrix}1&1&1&1\\1&2&3&4\end{pmatrix}\begin{pmatrix}1\\4\\6\\9\end{pmatrix}$$

$$=\frac{1}{20}\begin{pmatrix}30&-10\\-10&4\end{pmatrix}\begin{pmatrix}20\\63\end{pmatrix}=\frac{1}{20}\begin{pmatrix}600-630\\-200+252\end{pmatrix}$$

$$=\frac{1}{20}\begin{pmatrix}-30\\52\end{pmatrix}=\begin{pmatrix}-3/2\\13/5\end{pmatrix}=\begin{pmatrix}-1.5\\2.6\end{pmatrix}$$

The line is

$$f(x)=-1.5+2.6x$$

Graph is easy.

⑧

a_i	1	2	3	4
b_i	1	4	6	9

$$\begin{pmatrix} 1 \\ 4 \\ 6 \\ 9 \end{pmatrix} = \underset{\overset{\shortparallel}{A}}{\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{pmatrix}} \begin{pmatrix} r_0 \\ r_1 \\ r_2 \end{pmatrix}$$

$$A^TA = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 16 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{pmatrix} = \begin{pmatrix} 4 & 10 & 30 \\ 10 & 30 & 100 \\ 30 & 100 & 354 \end{pmatrix}$$

$$(A^TA)^{-1} = \frac{1}{80} \begin{pmatrix} 620 & -540 & 100 \\ -540 & 516 & -100 \\ 100 & -100 & 20 \end{pmatrix}.$$

So $\begin{pmatrix} r_0 \\ r_1 \\ r_2 \end{pmatrix} = (A^TA)^{-1} A^T \begin{pmatrix} 1 \\ 4 \\ 6 \\ 9 \end{pmatrix} = \begin{pmatrix} -1.5 \\ 2.6 \\ 0 \end{pmatrix}$

The least squares fit is

$$f(x) = -1.5 + 2.6x + 0x^2 = -1.5 + 2.6x$$

⑬

a_i	1	2	3	4	5
b_i	8	8	6	5	6

Using the idea of "x-symmetry" let $t_i = a_i - 3$. Then

t_i	-2	-1	0	1	2
b_i	8	8	6	5	6

Let

$$A = \begin{pmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{pmatrix} \Rightarrow A^TA = \begin{pmatrix} 5 & 0 \\ 0 & 10 \end{pmatrix}.$$

Then

$$\begin{pmatrix} r_0 \\ r_1 \end{pmatrix} = (A^TA)^{-1}A^T \begin{pmatrix} 8 \\ 8 \\ 6 \\ 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 6.6 \\ -0.7 \end{pmatrix}$$

So $f(t) = 6.6 - 0.7(t)$

$\Rightarrow f(x) = 6.6 - 0.7(x-3) = 6.6 + 2.1 - 0.7x$

$= 8.7 - 0.7x$

$f(6) = 8.7 - 4.2 = 4.5$

7.1) ③ $(-4)(x)+(-2)(x^2-1)+(\ 1\ )(x^3)+(\frac{5}{2})(2x^2)=x^3+3x^2-4x+2$

So the coord. vector is $(-4,-2,1,\frac{5}{2})$

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④ $(\frac{1}{2})(1)+(\frac{1}{2})(2x-1)+(\ 1\ )(x^3+x^4)+(\frac{-1}{2})(2x^3)+(0)(x^2+2)$

$=x+x^4$

So the coord. vector is $(\frac{1}{2},\frac{1}{2},1,\frac{-1}{2},0)$

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⑧

$$C_{E,B}=\begin{pmatrix}3&1&2\\4&1&2\\-1&2&1\end{pmatrix}\qquad v=\begin{pmatrix}2\\5\\-1\end{pmatrix}$$

$$v_B=C_{E,B}\,v=\begin{pmatrix}6+5-2\\8+5-2\\-2+10-1\end{pmatrix}=\begin{pmatrix}9\\11\\7\end{pmatrix}$$

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⑫ $B=\{(1,0,1),(1,1,0),(0,1,1)\}$

$B'=\{(0,1,1),(1,1,0),(1,0,1)\}$

ⓐ $B\rightarrow B'$

$$\left(\begin{array}{ccc|ccc}0&1&1&1&1&0\\1&1&0&0&1&1\\1&0&1&1&0&1\end{array}\right)\overset{R_1\leftrightarrows R_3}{\sim}\left(\begin{array}{ccc|ccc}1&0&1&1&0&1\\1&1&0&0&1&1\\0&1&1&1&1&0\end{array}\right)$$

$$\overset{R_2-R_1}{\sim}\left(\begin{array}{ccc|ccc}1&0&1&1&0&1\\0&1&-1&-1&1&0\\0&1&1&1&1&0\end{array}\right)\overset{R_3-R_2}{\sim}\left(\begin{array}{ccc|ccc}1&0&1&1&0&1\\0&1&-1&-1&1&0\\0&0&2&2&0&0\end{array}\right)$$

$$\overset{\frac{1}{2}R_3}{\sim}\left(\begin{array}{ccc|ccc}1&0&1&1&0&1\\0&1&-1&-1&1&0\\0&0&1&1&0&0\end{array}\right)\underset{R_1-R_3}{\overset{R_2+R_3}{\sim}}\left(\begin{array}{c|ccc}&0&0&1\\I&0&1&0\\&1&0&0\end{array}\right)\Rightarrow C_{B,B'}=\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$$

ⓑ $\mathcal{B}' \rightarrow \mathcal{B}$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & -1 & 1 & 1 & -1 & 0 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 2 & 2 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right) \Rightarrow C_{\mathcal{B}',\mathcal{B}} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Since $C_{\mathcal{B},\mathcal{B}'} = C_{\mathcal{B}',\mathcal{B}}$ we only need to check

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad ✓$$

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⑱ $V = P_3 \quad \mathcal{B} = \{x, x^2-1, x^3, 2x^2\}, \ \mathcal{B}' = \{x^3, x^2, x, 1\}$

$\mathcal{B} \rightarrow \mathcal{B}'$

$(x)_{\mathcal{B}'} = (0,0,1,0), \ (x^2-1)_{\mathcal{B}'} = (0,1,0,-1)$

$(x^3)_{\mathcal{B}'} = (1,0,0,0), \ (2x^2)_{\mathcal{B}'} = (0,2,0,0)$

So:

$$C_{\mathcal{B},\mathcal{B}'} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$