HW #8

8.3 # 2, 8, 17, 18

9.1 # 2, 8, 10, 17dg, 18

8.3) (2) $g(x,y) = 8 - (2x^2 - 8xy + 3y^2) + (2x^2y - y^3)$

$f_2(x,y) = -2x^2 + 8xy - 3y^2$

$$A = \begin{pmatrix} -2 & 4 \\ 4 & -3 \end{pmatrix}$$

$$|A - \lambda I| = (-2-\lambda)(-3-\lambda) - 16 = \lambda^2 + 5\lambda - 10$$

$$\Rightarrow \lambda = \frac{-5 \pm \sqrt{25+40}}{2} = \frac{-5 \pm \sqrt{65}}{2}$$

E. vals have opposite sign so no local extrema @ (0,0).

(8) $g(x,y) = 4 - (x^2 - 6xy + 9y^2) + (x^3 - y^3) + \dots$

$f_2(x,y) = -x^2 + 6xy - 9y^2$

$$A = \begin{pmatrix} -1 & 3 \\ 3 & -9 \end{pmatrix}$$

$$|A - \lambda I| = (-1-\lambda)(-9-\lambda) - 9 = \lambda^2 + 10\lambda = 0$$

$$\lambda = 0, -10$$

Since $\lambda_1 = 0$, the higher order terms play a role in the local extrema of g at $\vec{0}$.

(17) $g(x,y) = 10 + y^2$

(18) $g(x,y) = -5 - y^2$

9.1) (2) (a) $z+w = 7+2i$, $zw = 10+3+(15-2)i = 13+13i$

(b) $z+w = 3+i$, $zw = 2+2+(-1+4)i = 4+3i$

(8) (a) $|\sqrt{3}-i| = \sqrt{3+1} = 2$

$\mathrm{Arg}(\sqrt{3}-i) = \arctan\left(\frac{-1}{\sqrt{3}}\right) = \frac{-\pi}{6}$

(b) $|-\sqrt{3}-i| = 4$

$\mathrm{Arg}(-\sqrt{3}-i) = \arctan\left(\frac{1}{\sqrt{3}}\right) - \pi = \frac{-5\pi}{6}$

(10) $(-\sqrt{3}+i)^6 = (2e^{i\frac{\pi}{6}})^6 = 2^6 e^{i\pi} = 64(-1) = -64$

(17) (d) F. $\sqrt{0} = 0$

(g) F. $(a+bi)(a-bi) = a^2+b^2 \in \mathbb{R} \quad \forall a,b \in \mathbb{R}$

(18) $8 = 8(\cos 2n\pi + i\sin 2n\pi) \quad \forall n \in \mathbb{Z}$

A cube root of 8 has modulus $\sqrt[3]{8} = 2$.
and is of the form

$$2\left(\cos\frac{2n\pi}{3} + i\sin\frac{2n\pi}{3}\right)$$

$n=0 : 2(1+i0) = 2$

$n=1 : 2\left(\frac{-1}{2} + i\frac{\sqrt{3}}{2}\right) = -1 + i\sqrt{3}$

$n=2 : 2\left(\frac{-1}{2} - i\frac{\sqrt{3}}{2}\right) = -1 - i\sqrt{3}$