Math 146C - Ordinary and Partial Differential Equations III
Quiz 1
April 7, 2011

Name: Key

1	2	Total
/5	/5	/10

Problem 1. *(5 points) Find the eigenvalues and eigenfunctions of the boundary value problem:*

$$\begin{cases} y'' + \lambda y = 0, \\ y(0) = 0,\ y'(\pi) = 0. \end{cases}$$

Problem 2. *(5 points) Find the Fourier series of the function*

$$f(x) = \sin x + \cos x.$$

① $y'' + \lambda y = 0$

ⓐ $\lambda < 0 \Rightarrow \lambda = -\mu^2 \ (\mu > 0) \Rightarrow y = c_1 e^{\mu x} + c_2 e^{-\mu x}$

$y(0) = c_1 + c_2 = 0$, $y'(\pi) = \mu c_1 e^{\mu\pi} - \mu c_2 e^{-\mu\pi} = 0$

$\Rightarrow c_1 = -c_2$ $\qquad \Rightarrow c_1 e^{\mu\pi} = c_2 e^{-\mu\pi}$

$\Rightarrow c_1 e^{\mu\pi} = -c_1 e^{-\mu\pi} \Rightarrow \cancel{(e^{\mu\pi})^2 = -1}$ or $\boxed{c_1 = 0} \Rightarrow c_2 = 0$.

no neg. e.vals.

ⓑ $\lambda = 0 \Rightarrow y = c_1 x + c_2$. $y(0) = c_2 = 0$, $y'(\pi) = c_1 = 0$ $\qquad$ $\lambda = 0$ not an e.val.

ⓒ $\lambda = \mu^2 \ (\mu > 0) \Rightarrow y = c_1 \cos \mu x + c_2 \sin \mu x$

$y(0) = c_1 = 0$, $y'(\pi) = c_2 \mu \cos \mu\pi = 0 \Rightarrow \mu = \frac{2n-1}{2}$, $n \in \mathbb{N}$

$\Rightarrow \boxed{\lambda_n = \left(\frac{2n-1}{2}\right)^2 \text{ e.vals } \ \& \ y_n = \sin\frac{(2n-1)x}{2} \text{ e.funcs, } n \in \mathbb{N}}$

② Since f is already a sum of sines and cosines, the Fourier series of f is f, i.e. $\mathcal{F}(f(x)) = f(x)$.